ENGG. PHYSICS

1ST & 2nd SEM

All Branch

Under SCTE&VT,Odisha

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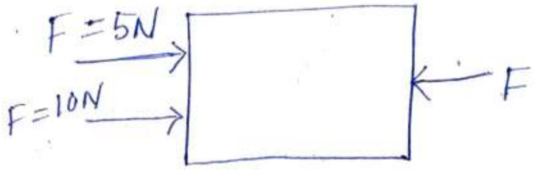
Scalar and vector

Physical quantity - The quantity which we can measure for that we need two thing Magnitude and direction.

→ some physical quantity have magnitude to represent but some physical quantity refers to both magnitude and direction.



here 5N or 10N represents a magnitude of physical quantity force

Physical quantity

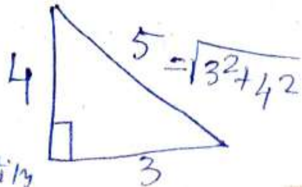
Scalar	Vector
(Scalars only Magnitude)	(Magnitudes as well as direction)
Mass, volume density, time	Force, displacement Velocity, acceleration

Scalar :- The physical quantity which can obey Algebric Addition is known as Scalar quantity

Add :- $5kg + 3kg = 8kg$ (Algebric Add^n)

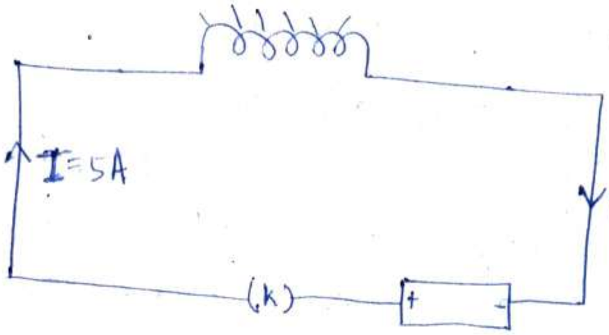
Vector Addition

The physical quantity which can obey Geometric addition known as vector quantity

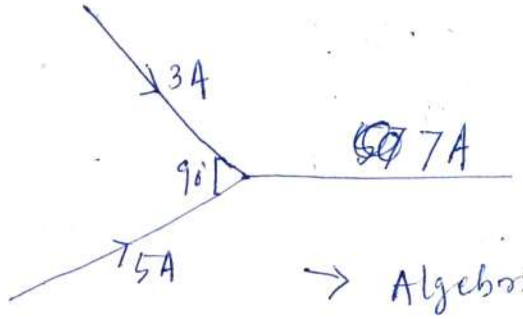


Magnitude :- Magnitude refers to a quantity or capacity or size. (e.g.- grand father slept)

Ex Current



both magnitude and direction



→ Algebric sum.

→ Current is that physical quantity which have both magnitude and direction but it obey Algebric sum. So we called it Tensor.

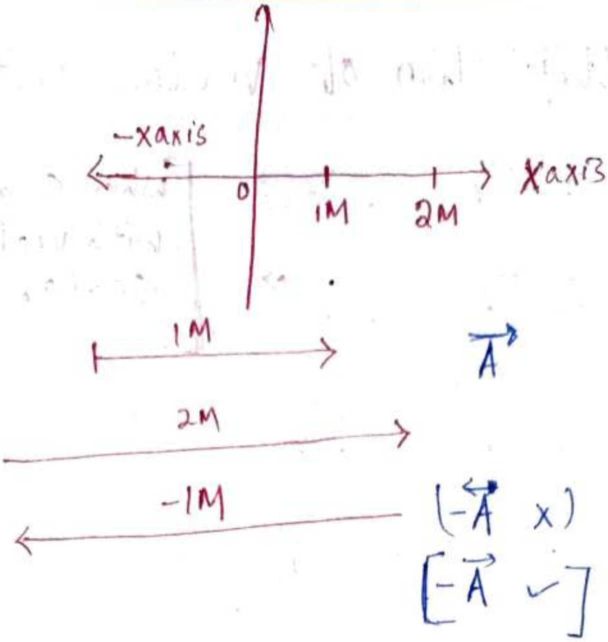
Tensor:- It is a scalar quantity which has direction.

Ex- Current, Pressure

Graphical representation of a vector :-

→ There is a sign to represent any physical quantity like $5°c$ → Temp. (vector →) represents

→ In vector (+ or -) represents their direction not their magnitude



$A \rightarrow$ Scalar quantity

$\vec{A} \rightarrow$ vector quantity

Problem-1

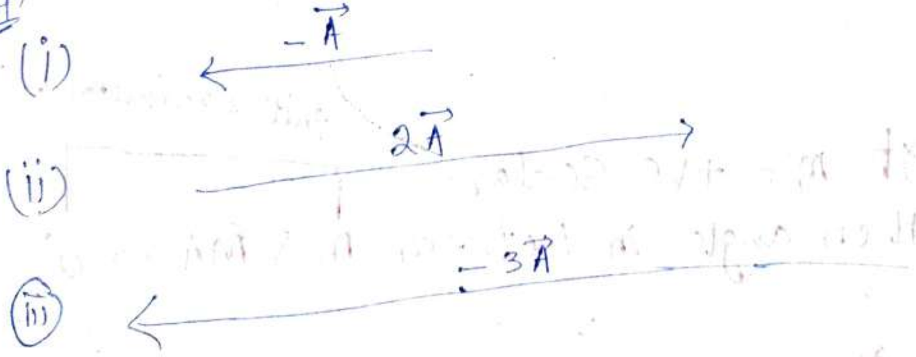
Draw the graphical representation of following physical quantity.

(i) $-\vec{A}$

(ii) $+2\vec{A}$

(iii) $-3\vec{A}$

Sol^n



Multipication of vector with a scalar

$m(\vec{A}) = m\vec{A}$

[When scalar multiply with vector it gives scalar]

ex-1 $2(\vec{A}) = 2\vec{A}$

(2) $-2(\vec{A}) = -2\vec{A}$

(3) $0(\vec{A}) = 0 \cdot \vec{A} = 0$

EX-1

$+\vec{A}$

$m = +2$

$m(\vec{A}) = +2\vec{A}$

[+ve scalar multiply with a vector then angle 0°]

EX-2

$m - 2$

$m(\vec{A}) = -2\vec{A}$

[-ve scalar multiply with +ve vector then angle is 180°.]

Note

i) If m = +ve scalar then angle in between m & $\underbrace{m\vec{A}}_{}$ is 0° (initial resultant)

ii) If m = -ve scalar then angle in between m & $m\vec{A}$ is 180°

Task

(i) If $m = +3$ then Draw $m\vec{A}$

(ii) If $m = -3$ then draw $\vec{mA}$

Types of vector

(i) Zero vector / ~~[illegible]~~ Null vector $(\vec{0})$
(ii) Unit Vector
(iii) orthogonal vector / Base vector
(iv) Equal vector
(v) Negative vector
(vi) co-linear vector
(vii) co-planer vector
(viii) Localised Vector
(ix) Non-localised vector

(i) Zero vector $(\vec{0})$:-

1) Magnitude $= 0$
2) It has no specific direction

Properties :-

① $\vec{A} + \vec{0} = \vec{A}$ (no change in magnitude & direc)

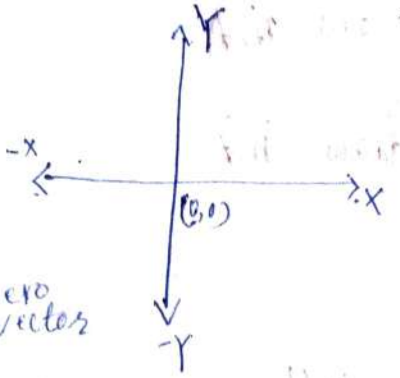
② $\vec{A} - \vec{0} = \vec{A}$

③ $\vec{A} \times \vec{0} = \vec{0}$

④ $\vec{A} \cdot \vec{0} = 0$

$\vec{A} - \vec{A} = \vec{0}$

EX



arbitrary direcn → zero vector

(ii) Unit vector :-

$\vec{A} = A\hat{A}$

Let's a student ask if $\vec{A}$ is a vector how to represent its magnitude & direction.

magnitude

Though only magnitude means scalar we don't put cap over A.

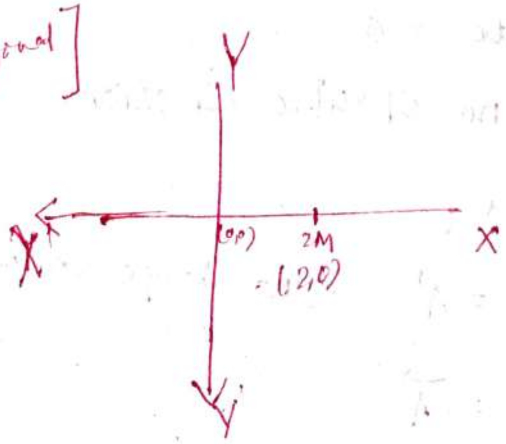
(i) $\hat{A}$ is a unit vector whose magnitude = 1

(ii) Its direction is along the direction of its parent vector.

ex $\vec{B} = B \cdot \hat{B}$ ($\hat{B}$ directed towards $\vec{B}$)

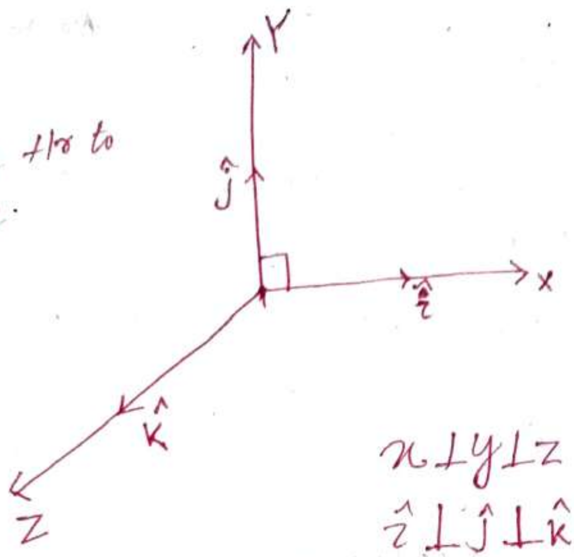
ex $\vec{C} = C \cdot \hat{C}$

Question [Orthogonal vector]



(iii) Orthogonal vector / Base vector :-

x, y, z are ⊥r to each other.



$x \perp y \perp z$

$\hat{i} \perp \hat{j} \perp \hat{k}$

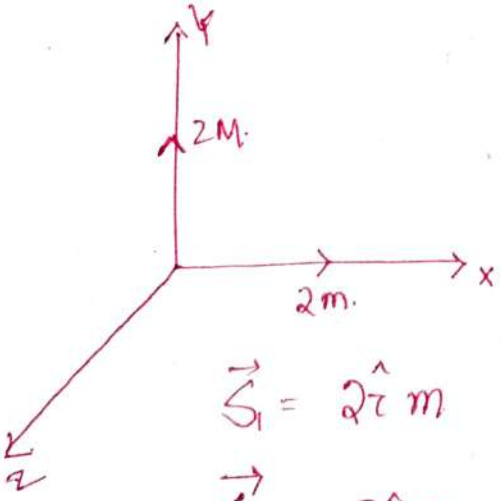
The unit vector which are ⊥r with each other is known as orthogonal vector.

ex- $\hat{i}, \hat{j}, \hat{k}$

→ All orthogonal vector bound to be unit vector

→ Unit vector could be orthogonal or not be orthogonal

EX



$\vec{S}_1 = 2\hat{i}\ m$

$\vec{S}_2 = 2\hat{j}\ m.$

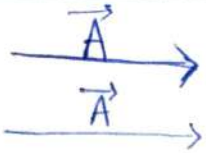
$\vec{S}_3 = 2\hat{k}\ m$

$\vec{S} = -2\hat{i}\ m$

magnitude = 2m

direction = -x axis

iv) Equal vector :-



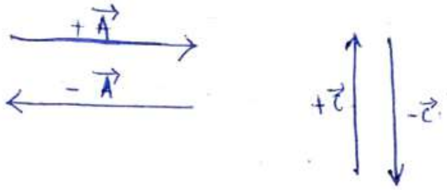
Length represent → magnitude
Arrow represent direction

Vector having same magnitude and same direction is known as Equal vector

→ Angle betⁿ them is zero (0°)

$$+\vec{A} = +\vec{C}$$

V) Negative vector :-

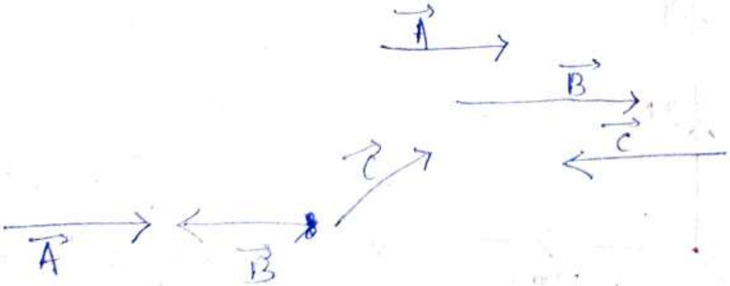


Same in magnitude but opposite in direction is known as Negative vector.

vi) Co linear vector :-

same

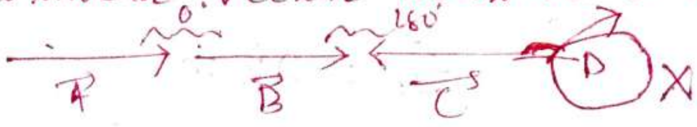
The vector having same line of action is called Co-linear vector.



here $\vec{A}$ & $\vec{B}$ are colinear but $\vec{C}$ is not as the line of action of $\vec{C}$ is not same

here condition

→ For colinear vector Must be 0° or 180°



→ If the angle between two vectors is not $0°$ or $180°$ then it is called Non. co-linear vector

ex.

Colinear vector

- Parallel vector

  $\theta = 0°$ (Angle bet^n vectors)

  Magnitude may or may not be equal.

- Anti parallel vector

  $\theta = 180°$

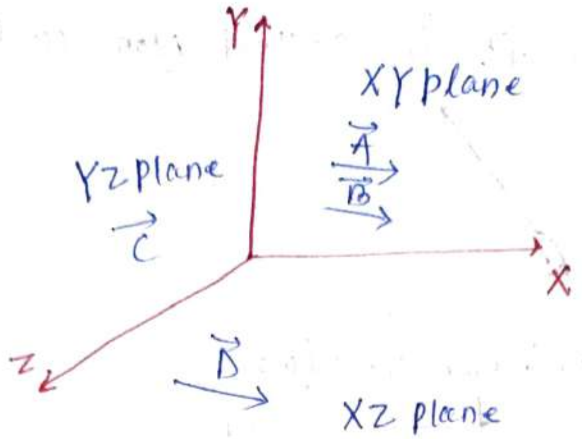
(*) What is the difference between equal and parallel vectors ?.

Ans In case of equal vectors magnitude ~~may~~ Should be same. but in case of parallel magnitude may be or may not be same.

→ All equal vectors are parallel but may or mayn't be parallel vectors are equal.

⇒ Incase of Anti parallel vector direction must be opposite magnitude may or mayn't be same but incase of Negative vector magnitude must be same and direction must be opposite.

vii) Coplaner vector :



→ Here $\vec{A}$ & $\vec{B}$ are coplaner vector.

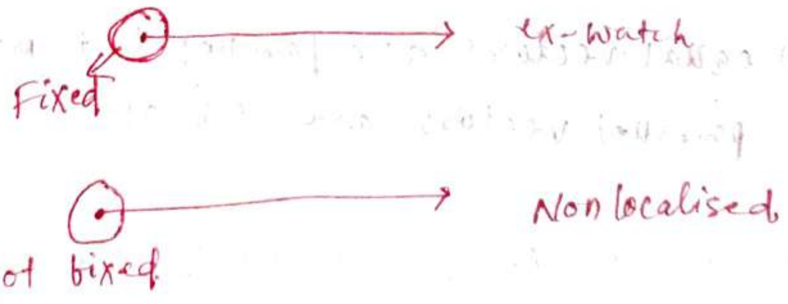
→ $\vec{A}$ & $\vec{C}$ or $\vec{C}$ & $\vec{D}$ can't be coplaner vector.

When two vectors lies on the same plane then it is said to be coplaner vector.

→ When ~~they are~~ vectors are not in same plane is called non-coplaner vector.

viii) Localised vector

Vector whose initial point is fixed is known as localised vector.



Home task

Write mathematical vector form

(1) 2m along $-x$ axis.

(2) 0.5m along z axis

Vector Addition

$$\vec{A} + \vec{B} = \vec{R}$$ (If we add two vector then the resultant must be a vector)

We have three methods for vector Addition.

(i) Triangle's law of vector Addition

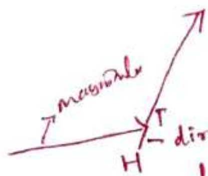
(ii) Parallelogram law of vector Addition

(iii) Polygon's law of vector Addition

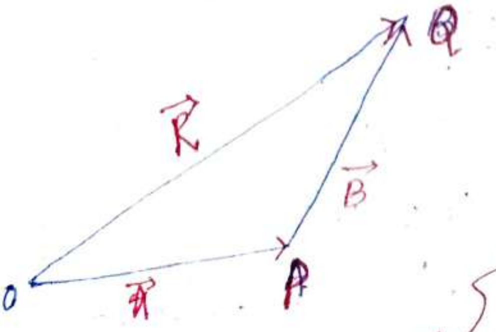
(i) Triangle's law

If two vectors are represents on the two sides of a triangle are taken in same order then their resultant vector must be represent on the third side of this triangle is taken in opposite order.

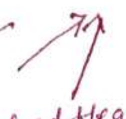
i.e $\vec{A} + \vec{B} = \vec{R}$



head-tail } same
tail-head } order

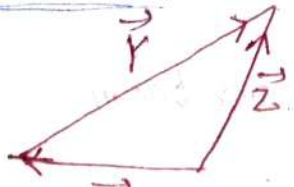


opposite order } Head-Head



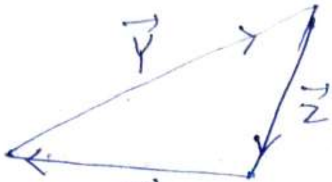
Graphical method

EX



$\vec{Z} = \vec{X} + \vec{Y}$

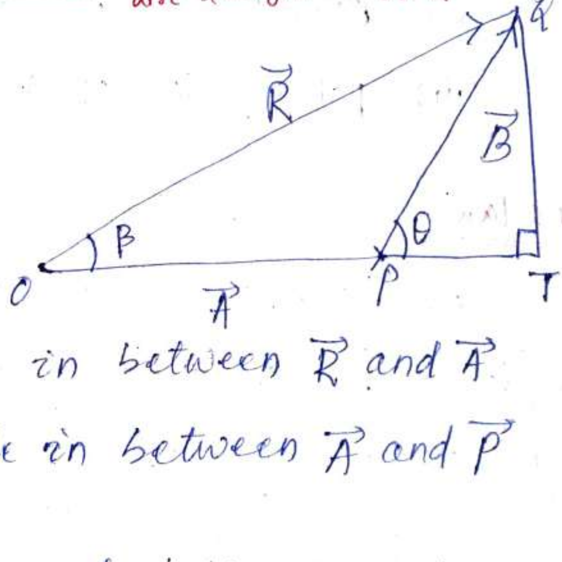
It is resultant vector as z & y are opposite order but x & y same order.



$\vec{X} + \vec{Y} + \vec{Z} = 0$ (Though their resultant vector is zero so it is a null vector)

Analytical method

In analytical method we will find 'dir' & magnitude of $\vec{R}$



β = angle in between $\vec{R}$ and $\vec{A}$

θ = angle in between $\vec{A}$ and $\vec{P}$

[→ we will taken angle between two vectors we want to add]

β → represents the direction of resultant vector $\vec{R}$.

From $\Delta\, OQT$

$\angle OTQ = 90^\circ$

According to Pythagoras $[h^2 = b^2 + p^2]$

$\Rightarrow OQ^2 = OT^2 + QT^2$

$\Rightarrow OQ^2 = (OP + PT)^2 + QT^2$

[as we are finding magnitude so i donot put ~~arrow~~ over any term.]

$$\Rightarrow OQ^2 = OP^2 + \underset{?}{PT^2} + 2 \cdot OP \cdot PT + \underset{?}{QT^2} \quad \text{---} (1)$$

Note [In ΔPTQ if h and angle(θ) known then to find base we use $\cos\theta$
to
& find perpendicular we use $\sin\theta$ we know]

Consider ΔPQT

$$\cos\theta = \frac{B}{H} = \frac{PT}{PQ} = \frac{PT}{B}$$

$$\Rightarrow \boxed{PT = B\cos\theta}$$

$$\sin\theta = \frac{P}{H} = \frac{QT}{PQ} = \frac{QT}{B}$$

$$\Rightarrow \boxed{QT = B\sin\theta}$$

Putting the values of PT & QT in eqn (1)

$$\Rightarrow R^2 = A^2 + B^2\cos^2\theta + 2AB\cos\theta + B^2\sin^2\theta$$

$$\Rightarrow R^2 = A^2 + B^2(\cos^2\theta + \sin^2\theta) + 2AB\cos\theta$$

$$\Rightarrow R^2 = A^2 + B^2 + 2AB\cos\theta$$

$$\Rightarrow \boxed{R = A^2 + B^2 + 2AB\cos\theta}$$

θ = Angle between $\vec{A}$ & $\vec{B}$ or two vector.

→ Whenever it is asked find the ~~value~~ direction of resultant vector then we have to find the value of β.

For direction of resultant vector

Consider Δ OTQ

$$\angle OTQ = 90^\circ$$

$$\tan\beta = \frac{P}{B} = \frac{QT}{OT} = \frac{QT}{OP+PT}$$

$$\Rightarrow \tan\beta = \frac{B\sin\theta}{A + B\cos\theta}$$

→ [I don't put vector sign over A & B Because we can add, subtract or multiply but we can't divide vector so i haven't put it.]

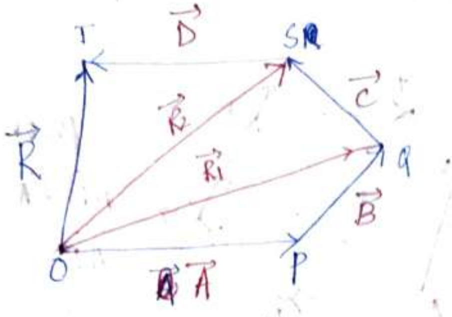
$$\Rightarrow \beta = \tan^{-1}\left[\frac{B\sin\theta}{A+B\cos\theta}\right]$$

β = direction of resultant vector.

(iii) Polygon's law :-

Statement - If no. of vectors are acting Simultaneously act at a point then the vectors are represent on the side of an open polygon are taken in same order then their resultant vector must be represent on the closing side of this polygon taken in opposite order.

$$\vec{A} + \vec{B} + \vec{C} + \vec{D} = \vec{R}$$



From Δ OPQ

$$\vec{R_1} = \vec{A} + \vec{B}$$

[As $\vec{A}$ and $\vec{B}$ are in same order but $\vec{R_1}$ in opposite order so it is a resultant vector]

From Δ OQS

$$\vec{R_2} = \vec{R_1} + \vec{C}$$

$$\vec{R_2} = (\vec{A} + \vec{B}) + \vec{C}$$

From Δ OST

$$\vec{R} = \vec{R_2} + \vec{D}$$

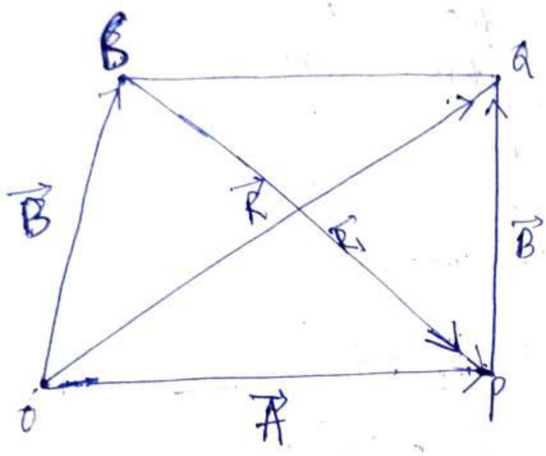
$$\rightarrow \boxed{\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}}$$

→ Polygon's law principle based upon triangles law

Parallelogram law of vector Addition :

Statement - If two vectors are represents on the two adjacent side of a parallelogram drawn from a common point then their resultant vector must be drawn from that point and it represent on the diagonal of this parallelogram

i.e $\vec{A} + \vec{B} = \vec{R}$



From Δ OPQ

$$\vec{OQ} = \vec{OP} + \vec{PQ}$$

$$\vec{R} = \vec{A} + \vec{B}$$

Δ OSP

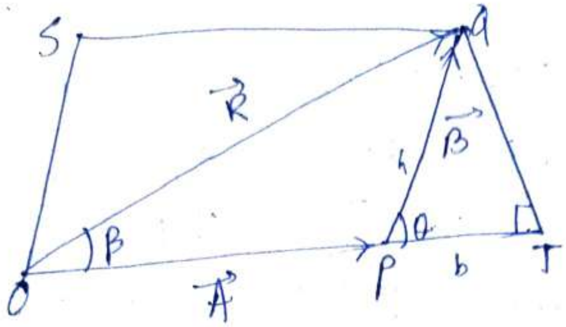
$$\vec{OP} = \vec{OS} + \vec{SP}$$

$$\Rightarrow \vec{A} = \vec{B} + \vec{R_1}$$

$$\Rightarrow \boxed{\vec{R_1} = \vec{A} - \vec{B}}$$

→ In parallelogram law of vector Addition if one diagonal is vector Add^n then another one must be the vector subtraction.

Analytical Method : (we will find magnitude & direc^n)



[one perpendicular drawn QT from point Q]

FROM Δ OQT

$$OQ^2 = OT^2 + QT^2$$

$$OQ^2 = (OP+PT)^2 + QT^2$$

$$\Rightarrow OQ^2 = OP^2 + PT^2 + 2\,OP\,PT + QT^2 \quad \text{—— (1)}$$

($\rightarrow$ not put bcz i find magnitude here)

FROM Δ PQT

$\angle PTQ = 90°$

$$\cos\theta = \frac{b}{h} = \frac{PT}{PQ} = \frac{PT}{B} \quad \Big| \quad \sin\theta = \frac{P}{h} = \frac{QT}{PQ} = \frac{QT}{B}$$

$$\Rightarrow QT = B\sin\theta$$

$$\Rightarrow PT = B\cos\theta$$

Put all these values in equation (1) becomes.

$$R^2 = A^2 + B^2\cos^2\theta + 2AB\cos\theta + B^2\sin^2\theta$$

$$\Rightarrow R^2 = A^2 + B^2\underbrace{(\cos^2\theta + \sin^2\theta)}_{1} + 2AB\cos\theta$$

$$\Rightarrow R^2 = A^2 + B^2 + 2AB\cos\theta$$

$$\Rightarrow R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

[θ = angle between 2 vectors]

Now we will find direction

From ΔOQT,

$\angle OTQ = 90^\circ$

$$\tan\beta = \frac{QT}{OT} = \frac{QT}{OP+PT}$$

$$\Rightarrow \tan\beta = \frac{B\sin\theta}{A+B\cos\theta}$$

Case-1 (Parallel vector)

$$\left.\begin{matrix} \xrightarrow{\quad\vec{A}\quad} \\ \xrightarrow{\quad\vec{B}\quad} \end{matrix}\right\} \theta = 0^\circ$$

Magnitude

$$R = \sqrt{A^2+B^2+2AB\cos\theta}$$
$$= \sqrt{A^2+B^2+2AB}$$
$$= \sqrt{(A+B)^2}$$
$$= A+B$$

Direction

$$\tan\beta = \frac{B\sin\theta}{A+B\cos\theta}$$
$$= \frac{B\sin 0^\circ}{A+B\cos 0^\circ}$$
$$= \frac{0}{A+B} = 0$$
$$\tan\beta = \tan 0^\circ$$
$$\beta = 0^\circ$$

→ In this case ~~magnitude and~~ direction is same as direction of two vector

Case-2 (Anti Parallel vector)

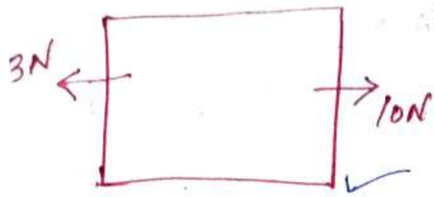
$$\left.\begin{matrix} \xrightarrow{\quad\vec{A}\quad} \\ \xleftarrow{\quad\vec{B}\quad} \end{matrix}\right\} \theta = 180^\circ$$

$$R = \sqrt{A^2+B^2+2AB\cos 180^\circ}$$
$$= \sqrt{A^2+B^2-2AB}$$
$$= \sqrt{(A-B)^2}$$
$$= A-B$$

→ In case of Antiparallel vector we subtract both the magnitude. Direction is towards the vector having more/much magnitude.

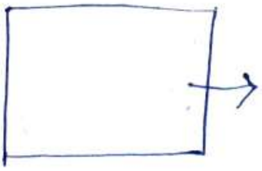
e.g

Find the net force.

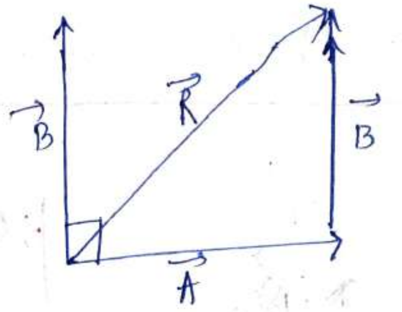


$= 10N - 3N$

$= 7N$ (∴ directed towards 10N)

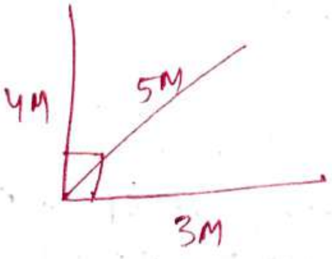


Case-3 Perpendicular vector



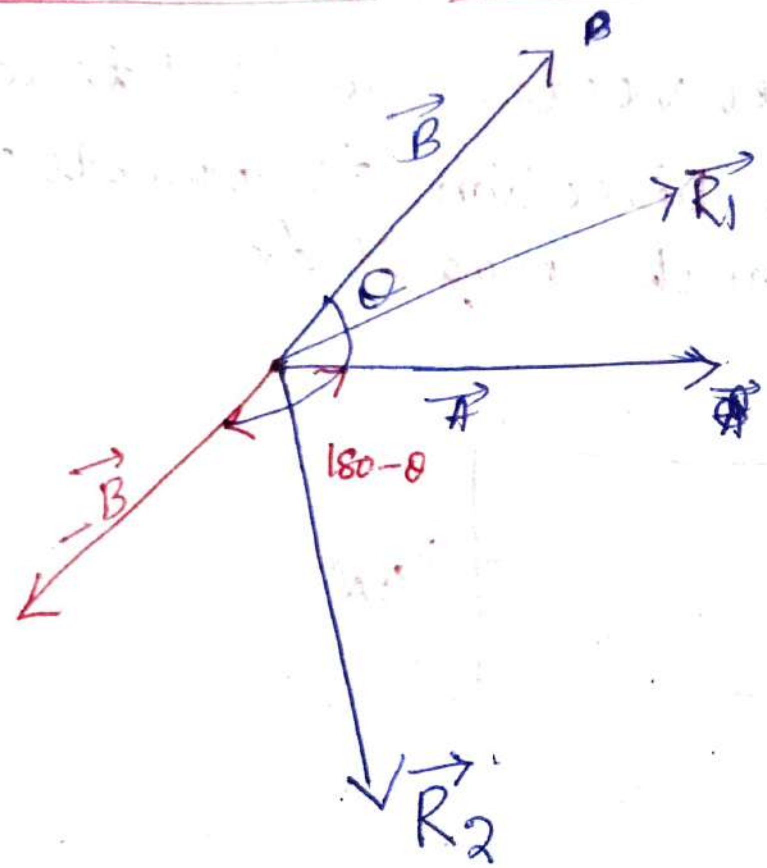
$$R = \sqrt{A^2 + B^2 + 2AB\cos 90^\circ}$$
$$= \sqrt{A^2 + B^2}$$

e.g



$$\sqrt{3^2 + 4^2} = 5$$

Vector subtraction



Here Angle = θ

$$\vec{R_1} = \vec{A} + \vec{B}$$

$$\therefore R_1 = \sqrt{A^2+B^2+2AB\cos\theta}$$

$$\vec{R_2} = \vec{A} + (-\vec{B})$$

$$\Rightarrow \vec{R_2} = \vec{A} - \vec{B}$$

Here Angle(θ) = 180 − θ

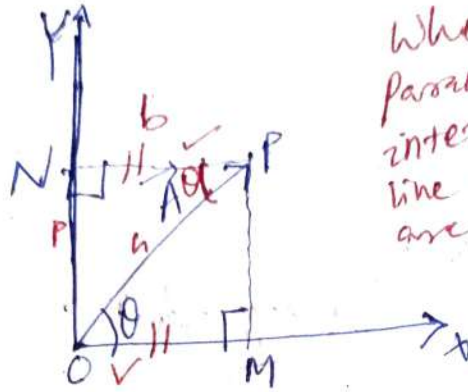
$$R_2 = \sqrt{A^2+B^2+2AB\cos(180-\theta)}$$

$$\boxed{R_2 = \sqrt{A^2+B^2-2AB\cos\theta}}$$

$[\cos(180-\theta) = -\cos\theta]$

Resolution of a vector :-

Spliting of a vector is called resolution of a vector



Whenever two parallel lines are intersect by third line the opposite angles are equal

In ΔOMP

$$\cos\theta = \frac{b}{h} = \frac{OM}{OP} = \frac{OM}{A}$$

[Sign not given over vector bcz we can't divide vector]

$\Rightarrow \boxed{OM = A\cos\theta}$ (For x-axis – $\cos\theta$)

In Δ OPN

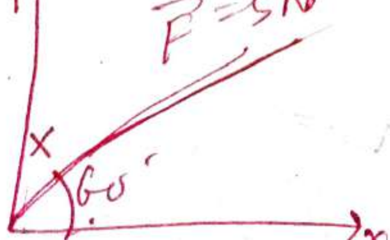
$$\sin\theta = \frac{p}{h} = \frac{ON}{OP} = \frac{ON}{A}$$

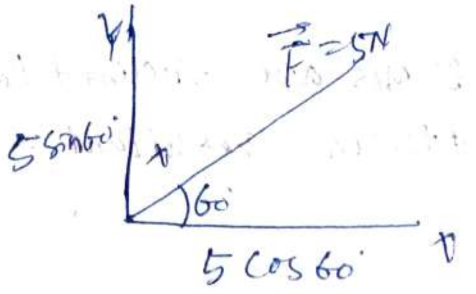
$\Rightarrow \boxed{ON = A\sin\theta}$ (For y-axis $\sin\theta$)

$A\cos\theta$ & $A\sin\theta$ are two component of $\vec{A}$

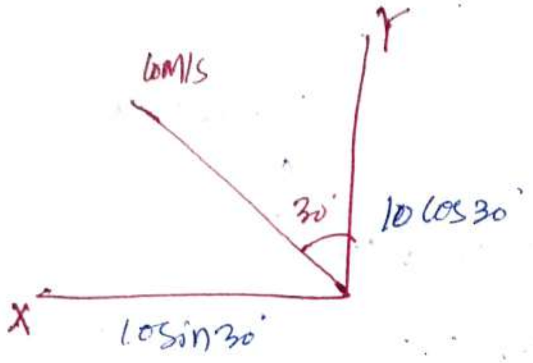
rectangular component

example





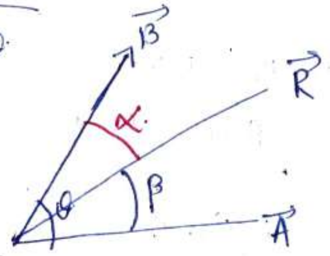
ex.



Formula required for problems on scalar & vector

(i) $\vec{R} = \vec{A} + \vec{B}$

$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$



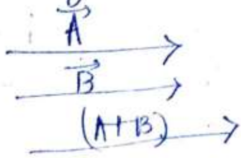
β → direcⁿ indicates

$\tan\beta = \dfrac{B\sin\theta}{A + B\cos\theta}$

Suppose angle α taken then

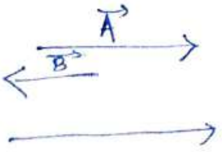
$\tan\alpha = \dfrac{A\sin\theta}{B + A\cos\theta}$

(ii) From parallelogram law of vector Addⁿ



→ Resultant. (Magnitude of two vector)

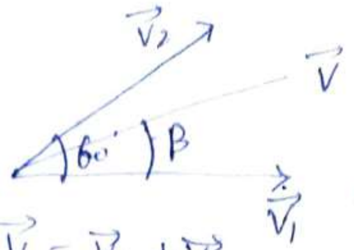
In Antiparallel vectors



In Antiparallel vector ~~resultant~~ two vector subtract to get Resultant and direcⁿ towards the vector having more magnitude

Q-1 Two velocities each 5 M/s are inclined to each other at 60° Find their resultant.

Sol)



$\vec{V} = \vec{V_1} + \vec{V_2}$

$V_1 = V_2 = 5 M/s$

$\theta = 60°$

Magnitude $V = \sqrt{V_1^2 + V_2^2 + 2V_1V_2\cos\theta}$

$= \sqrt{5^2 + 5^2 + 2.5.5.\cos 60}$

$= \sqrt{5^2 + 5^2 + \cancel{2}.5^2 . \frac{1}{\cancel{2}}}$

$= \sqrt{3 \times 5^2}$

$= 5\sqrt{3}$ M/s

$= 5 \times 1.732$

$= 8.660$ M/s

direction $\tan\beta = \dfrac{V_2 \sin\theta}{V_1 + V_2\cos\theta} = \dfrac{5\sin 60°}{5 + 5\cos 60°}$

$= \dfrac{\cancel{5}(\sin 60°)}{\cancel{5}(1 + \cos 60°)}$

$= \dfrac{\sin 60°}{1 + \cos 60°}$

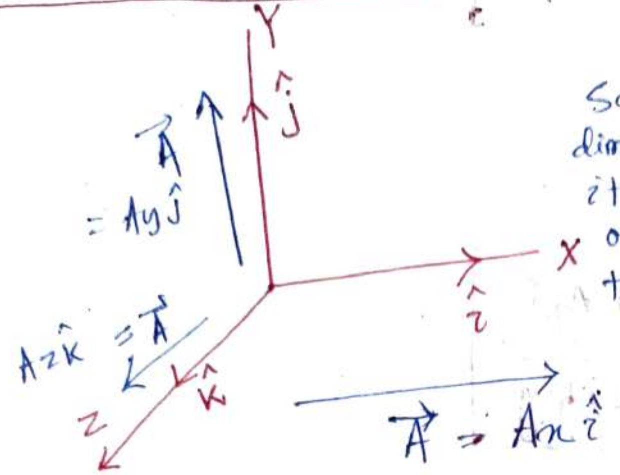
$\tan\beta = \dfrac{\frac{\sqrt{3}}{2}}{1 + \frac{1}{2}} = \dfrac{\frac{\sqrt{3}}{2}}{\left(\frac{3}{2}\right)} = \dfrac{\sqrt{3}}{\cancel{2}} \times \dfrac{\cancel{2}}{3}$

$= \dfrac{\sqrt{3}}{3} = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \tan\beta = \tan 30$

$\boxed{\beta = 30°}$

Vector in one dimension



A vector is said to be one ~~dimensi~~ dimensional vector if it is parallel with any one of axis of a three co-ordinate axis.

e.g. $\longrightarrow$ X

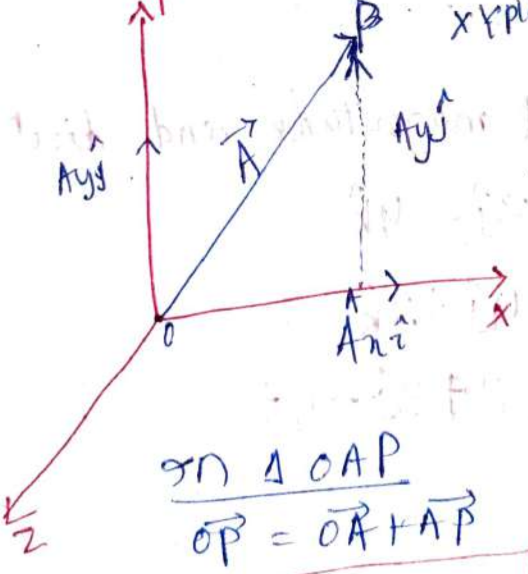
$\longrightarrow$

F = 5N.

$\vec{F} = 5\hat{i}$

Vector in two dimension

A vector is said to be two dimensional vector if it is lying in a two dimensional plane.



In Δ OAP

$\vec{OP} = \vec{OA} + \vec{AP}$

$\Rightarrow \boxed{\vec{A} = A_x\hat{i} + A_y\hat{j}}$

here $\vec{A}$ having two unit vector $\hat{i}$ & $\hat{j}$. So it is said to be two dimensional.

$$\boxed{|\vec{A}| = A = \sqrt{A_x^2 + A_y^2}}$$

$\because \vec{A} = A \cdot \hat{A}$

$\Rightarrow \hat{A} = \frac{\vec{A}}{A}$

$$\Rightarrow \boxed{\hat{A} = \frac{A_x\hat{i} + A_y\hat{j}}{\sqrt{A_x^2 + A_y^2}}}$$

e.g. $\vec{A} = 3\hat{i} + 4\hat{j}$

Find its magnitude and direction

Soln

$|\vec{A}| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$

$\hat{A} = \frac{3\hat{i} + 4\hat{j}}{5}$

$\hat{A} = \left(\frac{3}{5}\right)\hat{i} + \frac{4}{5}\hat{j}$

H.W

Question Find magnitude and dirn of following vecto

1. $\vec{A} = 3\hat{j} - 4\hat{k}$
2. $\vec{B} = 4\hat{j} + 2\hat{k}$
3. $\vec{C} = 2\hat{i} + 3\hat{j} - 4\hat{i}$