ENGINEERING MATHEMATICS-I

1st SEM

MECHANICAL ENGG.

Under SCTE&VT,Odisha

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4.1. MATRIX AND ITS ORDER

INTRODUCTION :

In modern engineering mathematics matrix theory is used in various areas. It has special relationship with systems of linear equations which occour in many engineering processes.

A matrix is a reactangular array of numbers arranged in rows (horizontal lines) and columns (vertical lines). If there are 'm' rows and 'n' Column's in a matrix, it is called a 'm' by 'n' matrix or a matrix of order m × n. The first letter in mxn denotes the number of rows and the second letter 'n' denotes the number of columns. Generally the capital letters of the alphabet are used to denote matrices and the actual matrix is enclosed in parantheses.

$$\text{Hence } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & -- & a_{1n} \\ a_{21} & a_{22} & a_{23} & -- & a_{2n} \\ a_{31} & a_{32} & a_{33} & -- & a_{3n} \\ -- & -- & -- & -- & -- \\ a_{m1} & a_{m2} & a_{m3} & -- & a_{mn} \end{bmatrix}$$

is a matrix of order m × n and 'a_{ij}' denotes the element in the ith row and jth column. For example a_{23} is the element in the 2nd row and third column. Thus the matrix 'A' may be written as (a_{ij}) where i takes values from 1 to m to represent row and j takes values from 1 to n to represent column.

If m = n, the matrix A is called a square matrix of order n × n (or simply n). Thus

$$A = \begin{bmatrix} a_{11} & a_{12} & -- & a_{1n} \\ a_{21} & a_{22} & -- & a_{2n} \\ a_{31} & a_{32} & -- & a_{3n} \\ & & -- & \\ a_{n1} & a_{n2} & -- & --a_{nn} \end{bmatrix}$$

is a square matrix of order n. The determinant of order n, $\begin{vmatrix} a_{11} & a_{12} & -- & a_{1n} \\ a_{21} & a_{22} & -- & a_{2n} \\ a_{31} & a_{32} & -- & a_{3n} \\ & & -- & \\ a_{n1} & a_{n2} & -- & --a_{nn} \end{vmatrix}$

which is associated with the matrix 'A' is called the determinant of the matrix and is denoted by det A or |A|.

Note : (1) Some times, a matrix A is denoted by [A], using brackets or || A || using double vertical bars.

(2) |A| should not be confused with the absolute value of A.

(iv) The matrix in which each element is the negative of the corresponding elements of a given matrix A is called the negative of A and denoted by (–A).

$$\text{Thus if } A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & 3 \\ 0 & 5 & -2 \end{bmatrix},$$

$$(-A) = \begin{bmatrix} -2 & 1 & -1 \\ 1 & -2 & -3 \\ 0 & -5 & 2 \end{bmatrix}$$

$$\text{Further } A + (-A) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

and $A + (-A) = (-A) + A$ i.e, $A + (-A) = 0 = (-A) + A$

The matrix (–A) is called the additive inverse of the matrix A.

(v) The substraction of two matrices A and B of the same order is defined as $A - B = A + (-B)$

$$\text{Thus if } A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 0 \\ 4 & -3 & -2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 1 & -3 \\ 0 & 5 & -1 \\ 3 & 4 & -2 \end{bmatrix} \text{ then } -B = \begin{bmatrix} -2 & -1 & 3 \\ 0 & -5 & 1 \\ -3 & -4 & 2 \end{bmatrix}$$

So that $A - B = A + (-B)$

$$= \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 0 \\ 4 & -3 & -2 \end{bmatrix} + \begin{bmatrix} -2 & -1 & 3 \\ 0 & -5 & 1 \\ -3 & -4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2 & 2-1 & -1+3 \\ 2+0 & 1-5 & 0+1 \\ 4-3 & -3-4 & -2+2 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 2 \\ 2 & -4 & 1 \\ 1 & -7 & 0 \end{bmatrix}$$

Product of a matrix and a scalar :

The product of a scalar m and a matrix A, denoted by mA, is the matrix each of whos elements is m times the corresponding element of A.

$$\text{Thus if } A = \begin{bmatrix} 2 & 1 & 3 \\ -1 & 0 & 4 \\ 3 & -2 & 1 \end{bmatrix}, \text{ then } 3A = \begin{bmatrix} 6 & 3 & 9 \\ -3 & 0 & 12 \\ 9 & -6 & 3 \end{bmatrix} \text{ and } 2A = \begin{bmatrix} 4 & 2 & 6 \\ -2 & 0 & 8 \\ 6 & -4 & 2 \end{bmatrix}$$

Note : The additive inverse of the matrix A is mA where $m = -1$

4.3. EQUALITY OF MATRICES

Two matrices A and B are said to be equal if and only if

(i) The orders of A is equal to that of B

(ii) Each element of A is equal to the corresponding element of B.

For example $\begin{pmatrix} a & b \\ x & y \end{pmatrix} = \begin{pmatrix} 1 & 5 \\ 0 & 6 \end{pmatrix}$

if and only if $a = 1, b = 5, x = 0, y = 6$

Addition of Matrices :

The sum of two matrices A and B is the matrix such that each of its elements is equal to the sum of the corresponding elements of A and B. The sum is denoted by A + B. Thus the addition of matrices is defined if they are of same order and is not defined when they are of different orders. A, B and (A + B) are of same order.

For example $\begin{pmatrix} a & b \\ x & y \end{pmatrix} + \begin{pmatrix} x & y \\ u & v \end{pmatrix} = \begin{pmatrix} a+x & b+y \\ x+u & y+v \end{pmatrix}$

Properties :

(i) The addition of a matrices is commutative, i.e. if A and B are two matrices of same order then $A + B = B + A$. From the definition of addition of matrices it follows that $A + B$ and $B + A$ are of some order. Further if $A = (a_{ij})$ and $B = (b_{ij})$

Then $A + B = (a_{ij} + b_{ij})$ and $B + A = (b_{ij} + a_{ij})$

But $(a_{ij} + b_{ij}) = (b_{ij} + a_{ij})$

i.e, each element of (A + B) is equal to the corresponding element of (B + A). Hence the result.

(ii) The matrix addition is associative i.e. if A, B and C are three matrices of same order, then $A + (B + C) = (A + B) + C$.

Since A, B and C are of same order,

$A + (B + C)$ and $(A + B) + C$ are of the same order.

Further if $A = (a_{ij}), B = (b_{ij}), C = (c_{ij})$

Then $A + (B + C) = a_{ij} + (b_{ij} + c_{ij})$

and $(A + B) + C = (a_{ij} + b_{ij}) + c_{ij}$

But $(a_{ij} + b_{ij}) + c_{ij} = a_{ij} + (b_{ij} + c_{ij})$

i.e, each element of $A + (B + C)$ is equal to the corresponding element of $(A + B) + C$. Hence the matrices $A + (B + C)$ and $(A + B) + C$ are equal i.e. the addition of matrices is associative.

(iii) The identity matrix for addition is the zero matrix or null matrix denoted by 0. Thus if A is a matrix, then $A + 0 = A$, provided the orders of the zero matrix is same as that of A.

Thus $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

and $(x\ y) + (0\ 0) = (x\ y)$

(iv) The matrix in which each element is the negative of the corresponding elements of a given matrix A is called the negative of A and denoted by (–A).

Thus if $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & 3 \\ 0 & 5 & -2 \end{bmatrix}$,

$$(-A) = \begin{bmatrix} -2 & 1 & -1 \\ 1 & -2 & -3 \\ 0 & -5 & 2 \end{bmatrix}$$

Further $A + (-A) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

and $A + (-A) = (-A) + A$ i.e, $A + (-A) = 0 = (-A) + A$

The matrix (–A) is called the additive inverse of the matrix A.

(v) The substraction of two matrices A and B of the same order is defined as $A - B = A + (-B)$

Thus if $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 0 \\ 4 & -3 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 & -3 \\ 0 & 5 & -1 \\ 3 & 4 & -2 \end{bmatrix}$ then $-B = \begin{bmatrix} -2 & -1 & 3 \\ 0 & -5 & 1 \\ -3 & -4 & 2 \end{bmatrix}$

So that $A - B = A + (-B)$

$$= \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 0 \\ 4 & -3 & -2 \end{bmatrix} + \begin{bmatrix} -2 & -1 & 3 \\ 0 & -5 & 1 \\ -3 & -4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2 & 2-1 & -1+3 \\ 2+0 & 1-5 & 0+1 \\ 4-3 & -3-4 & -2+2 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 2 \\ 2 & -4 & 1 \\ 1 & -7 & 0 \end{bmatrix}$$

oduct of a matrix and a scalar :

The product of a scalar m and a matrix A, denoted by mA, is the matrix each of whos elements is m times the corresponding element of A.

Thus if $A = \begin{bmatrix} 2 & 1 & 3 \\ -1 & 0 & 4 \\ 3 & -2 & 1 \end{bmatrix}$ then $3A = \begin{bmatrix} 6 & 3 & 9 \\ -3 & 0 & 12 \\ 9 & -6 & 3 \end{bmatrix}$ and $2A = \begin{bmatrix} 4 & 2 & 6 \\ -2 & 0 & 8 \\ 6 & -4 & 2 \end{bmatrix}$

***te* :** The additive inverse of the matrix A is mA where $m = -1$

Definition : The product of two matrices A and B (where the number of columns in A the n[umber] of rows in B) is the matrix AB whose element in the ith row and jth column is th[e sum] of the products formed by multiplying each element in the ith row of A a[nd the] corresponding element in the jth column of B.

Let 'A' be an $m \times k$ matrix and 'B' be a $k \times n$ matrix. The product of A and B, de[noted] by AB, is the $m \times n$ matrix with (i, j)th entry equal to the sum of the produc[ts of] corresponding elements from ith row of A and jth column of B. In otherwo[rds] $AB = [C_{ij}]$, then $C_{ij} = a_{i1} b_{1j} + a_{i2} b_{2j} + \ldots\ldots + a_{ik} b_{kj}$

For example (1) $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $B = \begin{bmatrix} x & y \\ u & v \end{bmatrix}$

$$AB = \begin{bmatrix} ax + bu & ay + bv \\ cx + du & cy + dv \end{bmatrix}$$

For example (2) $A = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ $B = [x \ y \ z]$, $AB = \begin{bmatrix} ax & ay & az \\ bx & by & bz \\ cx & cy & cz \end{bmatrix}$

for example (3) $A = [a \ b \ c]$ $B = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $AB = [ax + by + cz]$

Note : (1) The matrix product AA is defined only when A is a square matrix and is denoted b[y] similarly $A^3 = AA^2 = A^2A$

(2) The rule to remember a matrix product is $(m \times n)$ matrix $(n \times p)$ matrix = (m[× p)] matrix.

Properties :

(i) The multiplication of matrices is not necessarily commutative i.e., if A and B are [...] matrices then $AB \neq BA$.

(ii) The multiplication of matrices is associative i.e., if A, B, C are three matrices [...] $(AB)C = A(BC)$, provided the products are defined.

Proof : Let $A = \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix}$, $B = \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix}$ and $C = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$

Then $AB = \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix} = \begin{pmatrix} a_1b_1 + a_2b_3 & a_1b_2 + a_2b_4 \\ a_3b_1 + a_4b_3 & a_3b_2 + a_4b_4 \end{pmatrix}$

$(AB)\,C = \begin{pmatrix} a_1b_1 + a_2b_3 & a_1b_2 + a_2b_4 \\ a_3b_1 + a_4b_3 & a_3b_2 + a_4b_4 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$

$$= \begin{pmatrix} a_1b_1c_1 + a_2b_3c_1 + & a_1b_2c_2 + a_2b_4c_2 \\ a_3b_1c_1 + a_4b_3c_1 + & a_3b_2c_2 + a_4b_4c_2 \end{pmatrix}$$

$$BC = \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} b_1c_1 + b_2c_2 \\ b_3c_1 + b_4c_2 \end{pmatrix}$$

$$A(BC) = \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} \begin{pmatrix} b_1c_1 + b_2c_2 \\ b_3c_1 + b_4c_2 \end{pmatrix}$$

$$= \begin{pmatrix} a_1b_1c_1 + a_1b_2c_2 + a_2b_3c_1 + a_2b_4c_2 \\ a_3b_1c_1 + a_3b_2c_2 + a_4b_3c_1 + a_4b_4c_2 \end{pmatrix}$$

Hence (AB) C = A (BC)

(iii) The identity matrix for multiplication for the set of all square matrices of a given order is the unit matrix of the same order.

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, a square matrix of order 2

Taking $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ it can easily shown that

$AI = A = IA$

$$AI = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} a \times 1 + b \times 0 & a \times 0 + b \times 1 \\ c \times 1 + d \times 0 & c \times 0 + d \times 1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$\therefore \quad AI = A = IA$

similarly if A is a square matrix of order 3.

Taking $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ it can be proved that $AI = A = IA$.

(iv) Let A and B be two matrices such that the product AB is defined. Then $A = 0$ or $B = 0$ or $A = 0 = B$ always implies that $AB = 0$. Conversely. $AB = 0$ does not always imply that $A = 0$ or $B = 0$ or $A = 0 = B$

(v) The cancellation law does not hold for matrix multiplication, i.e. $CA = CB$ does not necessarily imply $A = B$,

(vi) The distributive laws hold for matrices, i.e., if A, B and C are three matrices then $A(B + C) = AB + AC$, $(A + B)C = AC + BC$ provided the addition and multiplication in above equations are defined.

4.4. THE MULTIPLICATIVE INVERSE OF A SQUARE MATRIX

If A and B are square matrices of order n such that AB = I = BA, where I is the identity matrix of order n then B is called the multiplicative inverse of A and is written as A^{-1}. Similarly A is called the multiplicative inverse of B and is written as B^{-1}.

Thus $AA^{-1} = I = A^{-1}A$, $B^{-1}B = I = BB^{-1}$

For example, the matrices $\begin{bmatrix} 4 & 3 \\ 5 & 4 \end{bmatrix}$ and $\begin{bmatrix} 4 & -3 \\ -5 & 4 \end{bmatrix}$ are multiplicative inverse of each other,

since $\begin{bmatrix} 4 & 3 \\ 5 & 4 \end{bmatrix}\begin{bmatrix} 4 & -3 \\ -5 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

and $\begin{bmatrix} 4 & -3 \\ -5 & 4 \end{bmatrix}\begin{bmatrix} 4 & 3 \\ 5 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

Note : 1. The zero matrix has no multiplicative inverse.

2. The unit matrix is the multiplicative inverse of itself

3. AB = BA does not necessarily imply that A and B are multiplicative inverse of each other.

For example, if $A = \begin{bmatrix} 0 & 3 \\ 3 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

Then $AB = BA = \begin{bmatrix} 0 & 9 \\ 9 & 0 \end{bmatrix} \neq I$

4. The inverse of A is denoted by A^{-1} (which is not equal to $\frac{1}{A}$)

***Example* : Find the multiplicative inverse of matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$**

Sol^n : Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $A^{-1} = \begin{pmatrix} e & f \\ g & h \end{pmatrix}$

Then $AA^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

or $\begin{pmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\therefore$ $ac + bg = 1$ $\qquad ce + dg = 0$

$af + bh = 0$ $\qquad cf + dh = 1$

Solving the above equation for e, f, g, h the following solutions are obtained

$$e = \frac{d}{ad - bc},\ f = \frac{-b}{ad - bc},\ g = \frac{-c}{ad - bc},\ h = \frac{a}{ad - bc}$$

Now, if $ad - bc \neq 0$, then

$$A^{-1} = \begin{bmatrix} \frac{d}{ad - bc} & \frac{-b}{ad - bc} \\ \frac{-c}{ad - bc} & \frac{a}{ad - bc} \end{bmatrix} = \frac{1}{ad - bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix},\ \text{where } |A| = \det. A$$

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

nspose :

Transpose of an $m \times n$ matrix A is the matrix of orders $n \times m$ obtained by interchanging the rows and columns of A. The transpose of the matrix A is written as A'.

For example if $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix},\ A' = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$

inors and Co-factors :

The minor of an element a_{ij} of a matrix is obtained by deleting the ith row and jth column from the matrix and is denoted by M_{ij}.

Thus if $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$

Then the minor of $a_{11} = M_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} = a_{22}a_{33} - a_{23}a_{32}$

the minor of $a_{21} = M_{21} = \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} = a_{12}a_{33} - a_{13}a_{32}$

the minor of $a_{32} = M_{32} = \begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} = a_{11}a_{23} - a_{13}a_{21}$ etc.

The co-factor of an element a_{ij} of a matrix A is $(-1)^{i+j} M_{ij}$ and is denoted by A_{ij}

Thus the co-factor of $a_{11} = A_{11} = (-1)^{1+1} M_{11} = (a_{22}a_{33} - a_{23}a_{32})$

the co-factor of $a_{21} = A_{21} = (-1)^{2+1}M_{21} = -(a_{12}a_{33} - a_{13}a_{32})$

The co-factor of $a_{32} = A_{32} = (-1)^{3+2}M_{32} = -a_{11}a_{23} + a_{13}a_{21}$ and so on.

Adjoint of a Matrix :

The adjoint of a matrix A is the transpose of the matrix obtained replacing each element in A by its cofactor A_{ij}. The adjoint of A is written as adj A. Thus if

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$\text{then adj } A = \begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix}$$

Example : **Find the adjoint of the matrix**

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 0 \\ -2 & 2 & -1 \end{bmatrix}$$

Solution : Now $A_{11} = (-1)^{1+1} M_{11} = \begin{vmatrix} 1 & 0 \\ 2 & -1 \end{vmatrix} = -1$

$$A_{12} = (-1)^{1+2} M_{12} = -\begin{vmatrix} 3 & 0 \\ -2 & -1 \end{vmatrix} = 3$$

$$A_{21} = (-1)^{2+1} M_{21} = -\begin{vmatrix} 2 & 3 \\ 2 & -1 \end{vmatrix} = 8$$

$$A_{13} = (-1)^{1+3} M_{13} = \begin{vmatrix} 3 & 1 \\ -2 & 2 \end{vmatrix} = 8$$

$$A_{22} = (-1)^{2+2} M_{22} = \begin{vmatrix} 1 & 3 \\ -2 & -1 \end{vmatrix} = 5$$

$$A_{23} = (-1)^{2+3} M_{23} = -\begin{vmatrix} 1 & 2 \\ -2 & 2 \end{vmatrix} = -6$$

$$A_{31} = (-1)^{3+1} M_{31} = \begin{vmatrix} 2 & 3 \\ 1 & 0 \end{vmatrix} = -3$$

$$A_{32} = (-1)^{3+2} M_{32} = -\begin{vmatrix} 1 & 3 \\ 3 & 0 \end{vmatrix} = 9$$

$$A_{33} = (-1)^{3+3} M_{33} = \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = -5$$

$\therefore$ Adjoint of A i.e., Adj A = the transpose of $[A_{ij}]_{3 \times 3}$

$$=\begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix} = \begin{pmatrix} -1 & 8 & -3 \\ 3 & 5 & 9 \\ 8 & -6 & -5 \end{pmatrix}$$

further $|A| = \det A = \begin{vmatrix} 1 & 2 & 3 \\ 3 & 1 & 0 \\ -2 & 2 & -1 \end{vmatrix}$

$= -1 - 2(-3-0) + 3(6+2)$
$= -1 + 6 + 24 = 29$

$$A(\text{adj } A) = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 0 \\ -2 & 2 & -1 \end{bmatrix}\begin{bmatrix} -1 & 8 & -3 \\ 3 & 5 & 9 \\ 8 & -6 & -5 \end{bmatrix} = \begin{bmatrix} 29 & 0 & 0 \\ 0 & 29 & 0 \\ 0 & 0 & 29 \end{bmatrix} = 29 \times I = |A|\, I$$

$$(\text{Adj}A)\, A = \begin{bmatrix} -1 & 8 & -3 \\ 3 & 5 & 9 \\ 8 & -6 & -5 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 0 \\ -2 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 29 & 0 & 0 \\ 0 & 29 & 0 \\ 0 & 0 & 29 \end{bmatrix} = 29 \times I = |A|\, I$$

$\therefore \quad A(\text{adj } A) = |A|\, I = (\text{adj } A)\, A$

or, $A\dfrac{\text{adj } A}{|A|} = I = \dfrac{\text{adj } A}{|A|}A$

From the above example it is clear that $\dfrac{\text{adj } A}{|A|}$ is the inverse of A.

Hence it can be stated that if A is a non-singular square matrix (i.e, $|A| \neq 0$) of order 3 then $A^{-1} = \dfrac{\text{adj } A}{|A|}$.

4.5. SOLUTION OF A SYSTEM OF LINEAR EQUATIONS BY MATRIX METHOD

Suppose we have the following system of equations

$a_1x + b_1y + c_1z = d_1$
$a_2x + b_2y + c_2z = d_2$
$a_3x + b_3y + c_3z = d_3$

Where $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$