DESIGN OF MACHINE ELEMENTS

TH-2

5th SEM

MECHANICAL ENGG.

Under SCTE&VT,Odisha

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Machine Design

→ It is also called as Design of machine element
→ Machine design is a subject which deals with the plan, construction and analysis of m/c elements.
→ Machine elements may be screw, bolt, nut, joints like (welding, rivets), gears, belt, rope, chains screw jack, spring etc

OR.
Machine design is the creation of new and better m/c and improving the existing one

Types of Design

It may be following three types

1. Adaptive Design
2. Development Design
3. New Design

(i) Adaptive Design

→ It is a design in which we make minor modification in the existing design or product
→ Only some basic technical skill required

Example.
Bicycle
Diesel engine

(ii) Development Design

→ In this case, the existing design or product are developed
→ Entire concept is new & developed to make a product better.

Example - electronic equipment or components like cellphone, Tape records, Television

(iii) New Design

-> This type of design needs lot of research, Technical Ability and creative thinking.

-> This is the most complicated design.

Example - Solar power plant, Tidal power plant, 3D printers

State the factors governing the design of m/c elements

1. Types of load
2. motion of parts
3. selection of material
4. size of the parts
5. Frictional Resistance and lubrication
6. convenient and economical features
7. use of standards parts
8. Safety operation
9. workshop facilities
10. number of m/c to be machined
11. cost construction
12. Assembling

Described Design procedure

(i) Need or Aim -: First of all we indicate the need i.e. aim, purpose for which the m/c is to be designed

(ii) Synthesis (mechanism) -:

Select the possible mechanism or group of mechanism which will give the desired motion

(iii) Analysis of force -.

-> Find the forces acting on the each member of the m/c

(iv) material selection -:

-> select the material which is best suitable for each member of the m/c

(v) Design of element size & stresses -:

Find the size of each member of the m/c by considering force acting on the member.

(vi) modification -: modify the shape and size of the product

Need or Aim
↓
Synthesis (mechanism)
↓
Analysis of force
↓
material selection
↓
Design of elements size and stresses
↓
modification
↓
Detailed Drawing
↓
production

(vii) Detailed Drawing -: Draw the detailed drawing of each component of the m/c with complete specification.

(viii) production - The component as per drawing is manufactured in the workshop.

State mechanical properties of material

① Strength -: It is the ability of a material to resist the external applied forces without breaking

② Stiffness(K) -: It is the ability of the material to resist deformation under the action of load

$$K = \frac{\text{load}\ (w)}{\text{Deflection}\ (\delta)}$$

③ Elasticity -: It is the property of material to regain its original position when load is removed

④ Plasticity -: It is the property of material which does not regain its original position when load is removed

⑤ Ductility -: It is the of the material by which the material can be drawn in wires with the application of tensile load

Example - mild steel, copper, aluminium, nickel, zinc, Tin, lead.

⑥ malleability -: It is the property of the material in which the material can be converted in to thin sheets with the application of compressive load.

Example - Steel, Brass, Bronze etc.

⑦ Brittleness -: It is the property of the material in which the material can be break or crack with little percentage of elongation

Example - wood, concrete

⑧ Toughness -: It is the property by which a material is able to resist shocks or impact loading without fracture.

⑨ Hardness -: It is the property of a material by which it can resist scratches, marks or wear and tear.

-> Brittle material are more hard

-> Hardness can be determined by following

(i) Brittle hardness Test

(ii) Rockwell " "

(iii) Vickers " "

⑩ Creep -: It is the property of the material to resist constant stress at high temp for long period of time. It will undergo slow and permanate deformation.

OR, It is the property of the material to resist high temp or extremly high temp.

(11) Fatigue :- It is the property of a material to resist repeatable and fluctuating load. It fails at stresses below the yield point stresses such type of failure of material is known as fatigue.

Example - Connecting Rod.

Define load and Types of load

load.

→ Any external force acting on the body is known as load.

Types.

① Dead load :- A load is said to be dead load. when it does not change in magnitude or direction.

② live load :- A load is said to be live load. when it changes continuously

③ Suddenly applied load :- A load is said to be suddenly applied load. when it is suddenly applied or removed

④ Impact load :- A load is said to be impact load. when it is applied with some initial velocity.

Working Stress or Design Stress

→ It is the ratio between maximum stress and factor of safety

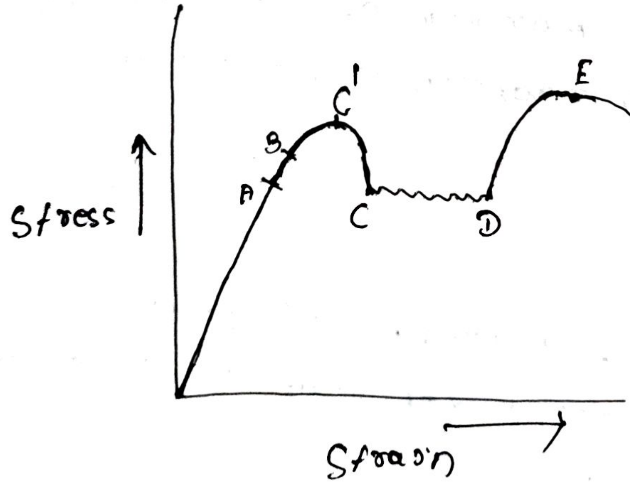
$$\text{working stress} = \frac{\text{maximum stress}}{\text{factor of safety}}$$

Factor of Safety

It is the ratio between maximum stress and working stress

$$\text{factor of safety} = \frac{\text{maximum stress}}{\text{working stress}}$$

Stress strain curve for mild steel



A = proportional limit
B = Elastic limit
C' = upper yield point
C = lower yield point
D = Strain hardening start
E = Ultimate Stress
F = fracture point

OA = proportional limit
OB = Elastic region
BC = Elasto plastic region
CD = perfectly plastic region
DE = Strain hardening
EF = necking region

(i) proportional limit (A)

→ in this limit the stress is directly proportional to strain. That means the steel rod obey's Hook's law

$$\sigma \propto e$$

$$\sigma = E e$$

σ = stress
e = strain
E = young modulus.

(ii) Elastic limit (B)

→ It is the point in the stress - strain curve upto which the materials remain elastic

→ upto this point there is no permanent deformation after removal of load

yield point.

→ This point is just beyond the elastic limit at which the material undergoes an appreciable increase in length without further increase in the load

→ There are two yield point
→ upper yield point C'
→ lower yield point C

(v) Strain hardening

→ in this range where additional stress produces additional strain.

(vi) Ultimate Stress(E) - The point at which the stress is maximum is knowns ultimate point. The stress corresponding to this point is known as ultimate stress.

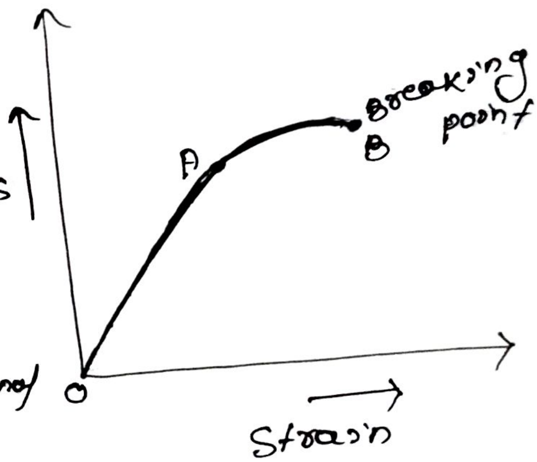
(vii) Fracture point or Breaking point (F)

→ The point at which the material fails or break is known as breaking point. stress corresponding to this point is known as breaking stress

Stress Strain curve of C.I

(i) A = Approximately a proportional limit

(ii) B = Breaking point



(i) A = Approximately proportional limit

→ In this limit stress is directly proportional to strain. It obey's Hook's law. OA is a straight line

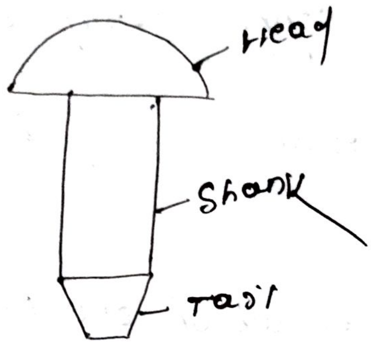
(ii) B = Breaking point -

→ The point at which the material is break is known as Breaking point. A to B both stress and strain increase.

-> Brittle materials develop internal crack first and then the cracks reached the surface.

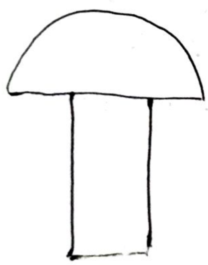
Rivet

- A rivet is a short cylindrical bar with a head integral to it
- It is used to join two or more material

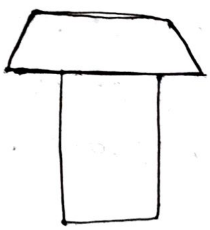


- Top portion of rivet is called head
- Middle portion of rivet is called shank
- Bottom portion of rivet is called tail

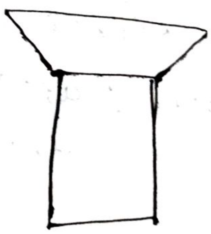
Types of Rivet head



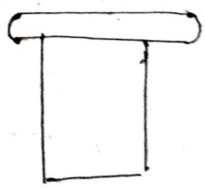
(Snap head)



(Pan Head)



(counter head)



(Flat head)

(Steeple head)

TYPES OF Riveted Joints

There are two TYPES

1. Lap Joint
2. Butt Joint
   - Single Strap butt Joint
   - Double Strap butt Joint

Lap Joint

→ A lap Joint is that in which one plate overlaps the other and the two plates are then riveted together.

Butt Joint

→ A butt joint is that in which the main plates are kept in touching each other and a cover plate (strap) is placed either on one side or on both sides of the main plates. The cover plate is then riveted together with the main plate.

Butt joints are following two types

1. Single Strap butt Joint
2. Double Strap butt Joint

Single Strap butt Joint

→ The edges of the main plates butt against each other and only one cover plate is placed on one side of the main plates and then riveted together.

Double Strap butt Joint

→ The edges of the main plates butt against each other and two cover plate is placed on both side of the main plate and then riveted together.

→ In addition to above, following are the types of riveted joints depending upon the number of rows of the rivets.

(i) Single riveted joint

(ii) Double riveted joint

Single Riveted Joint

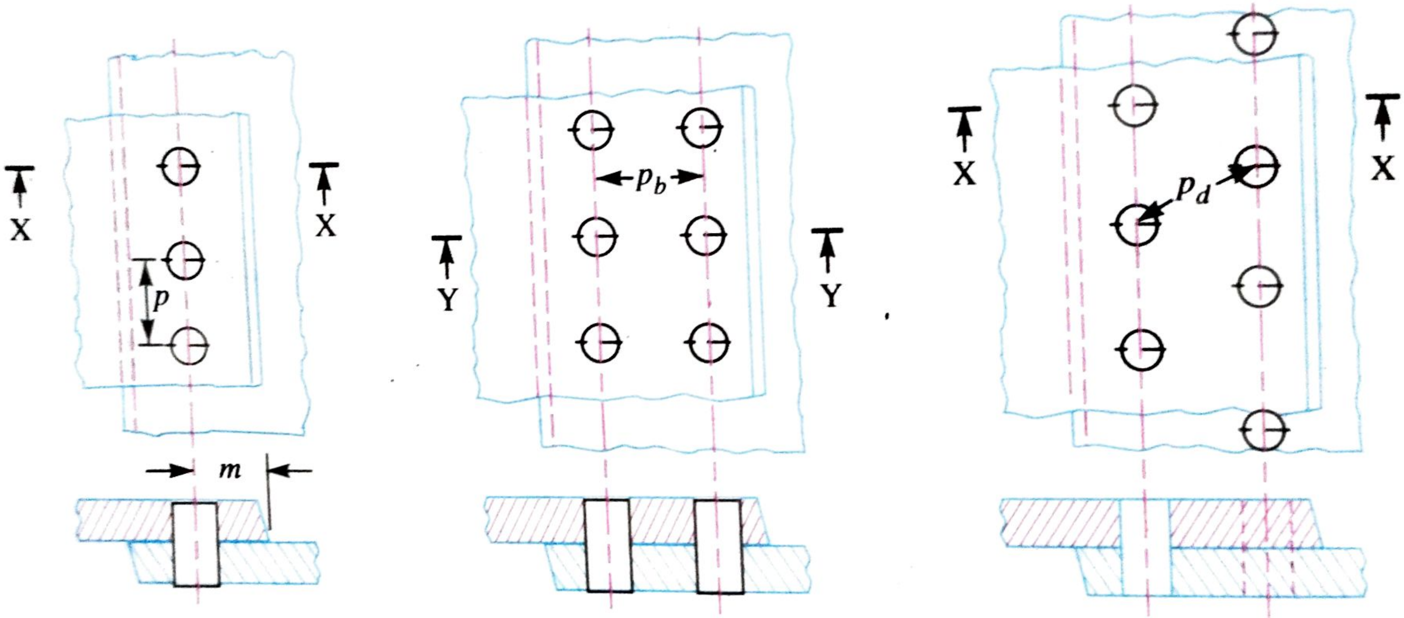
→ A single Riveted Joint is that in which there is a single row of rivets in a lap joint

→ A single Riveted Joint is that in which there is a single row of rivets on each side in a butt joint.

Double riveted joint

→ A double riveted joint is that in which there are two rows of rivets in a lap joint

→ A double riveted joint is that in which there are two rows of rivets on each side in a butt joint

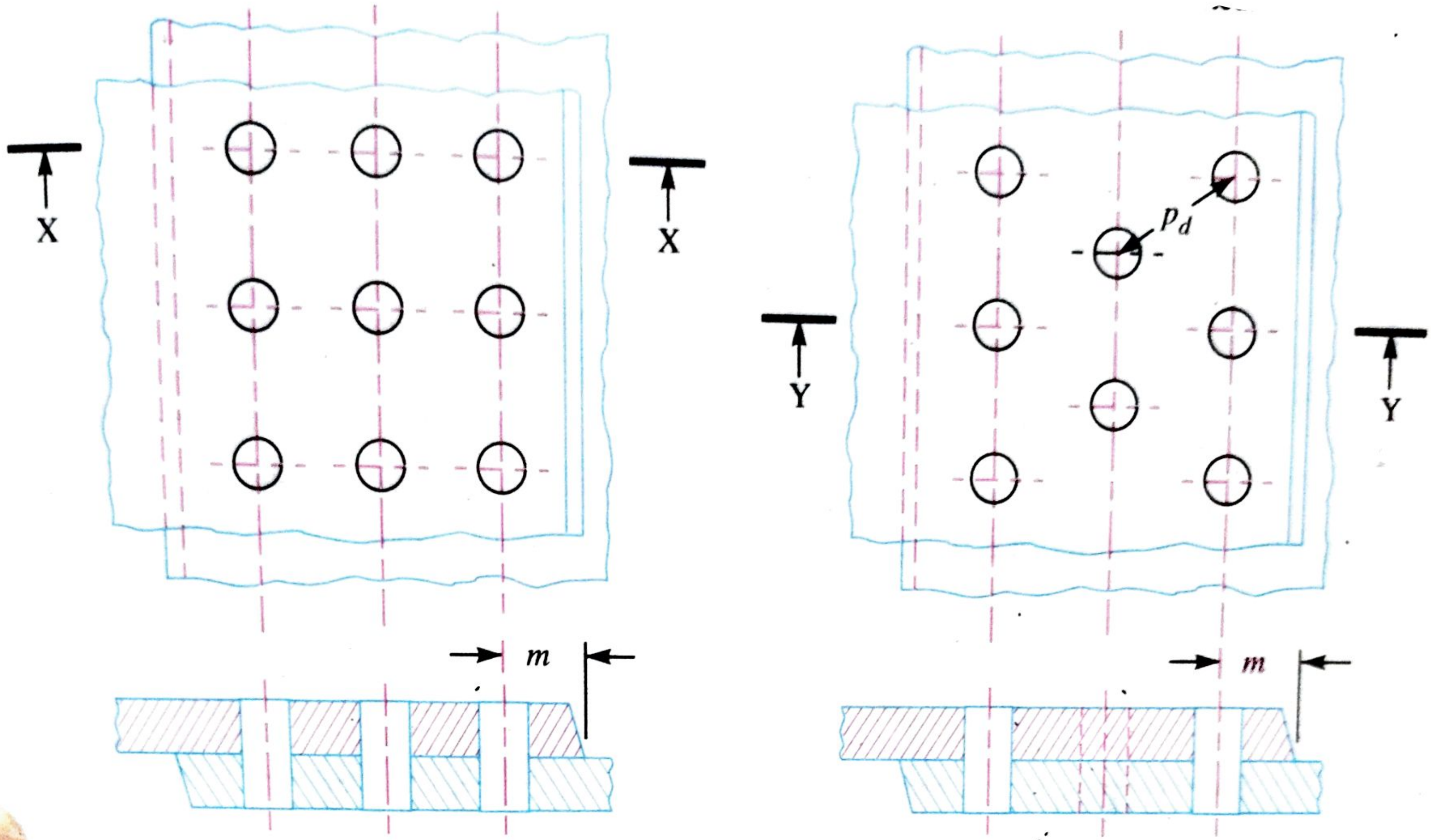


(*a*) Single riveted lap joint.

(*b*) Double riveted lap joint (Chain riveting).

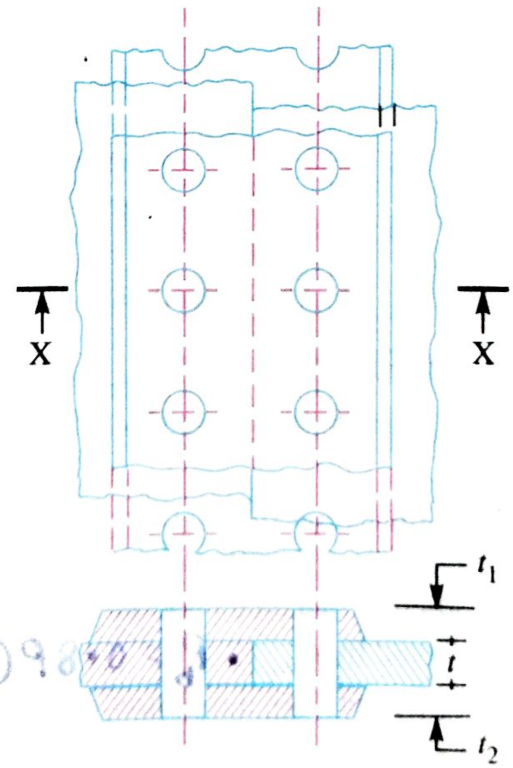
(*c*) Double riveted lap joint (Zig-zag riveting).

Single and double riveted lap joints.

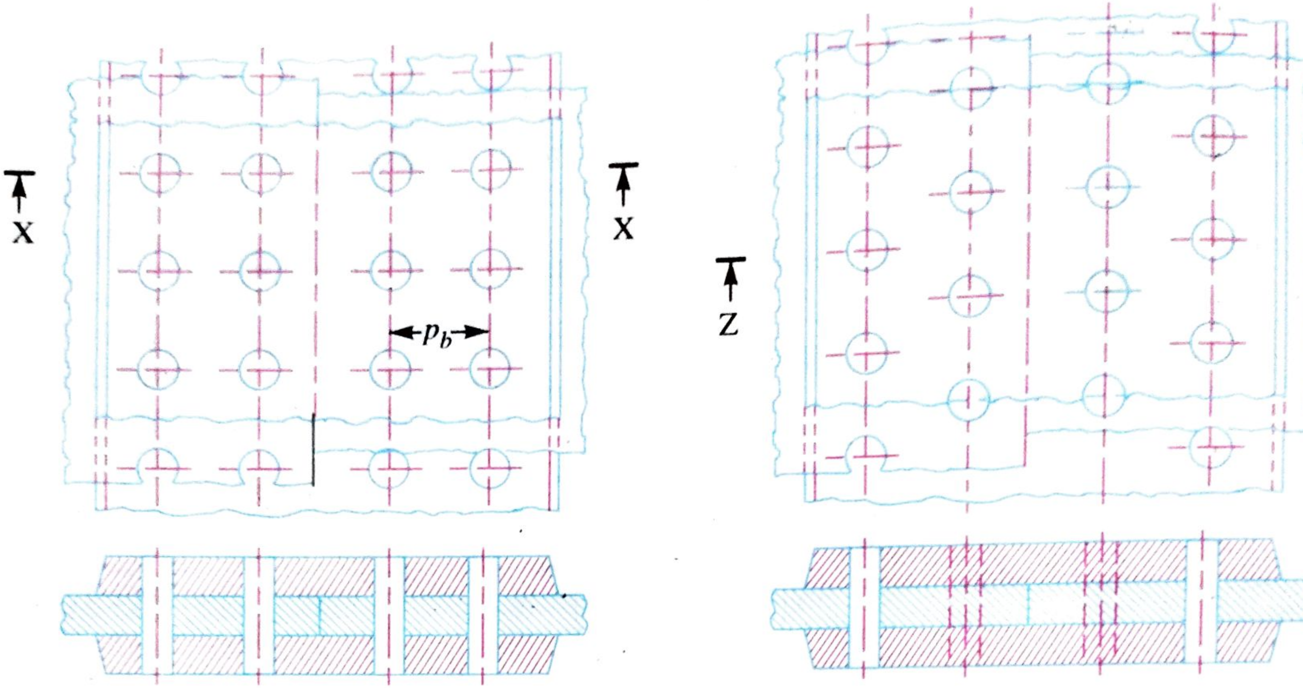


(*a*) Chain riveting. (*b*) Zig-zag riveting.

Triple riveted lap joint.



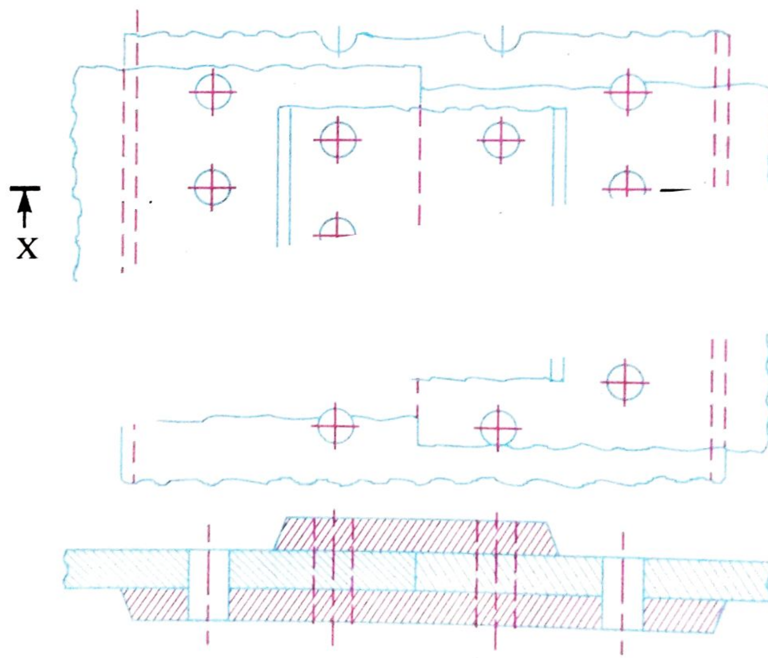
Single riveted double strap butt joint.



(*a*) Chain riveting.

(*b*) Zig-zag riveting.

Double riveted double strap (equal) butt joints.



9. Double riveted double strap (unequal) butt joint with zig-zag riveting.

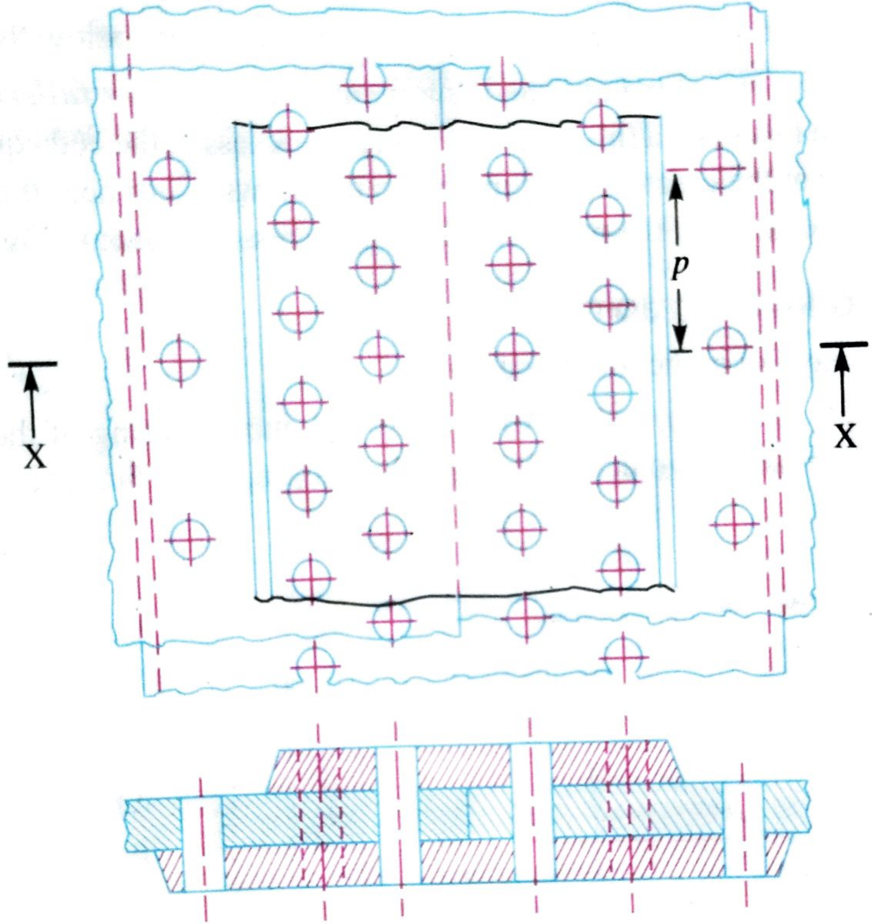
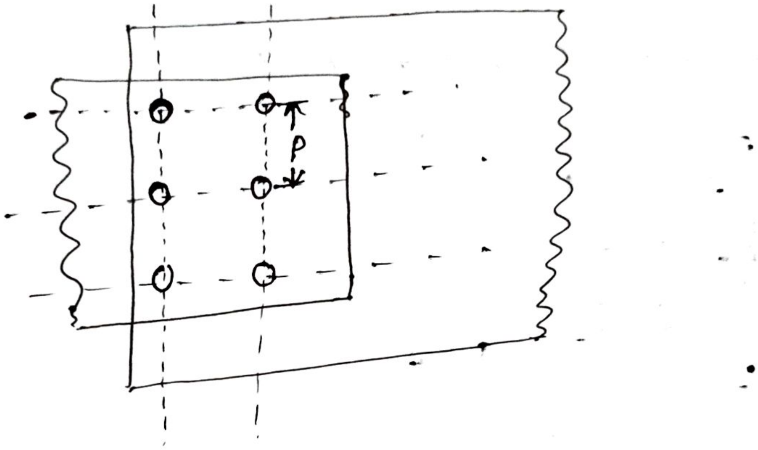


Fig. 9.11. Triple riveted double strap (unequal) butt joint.

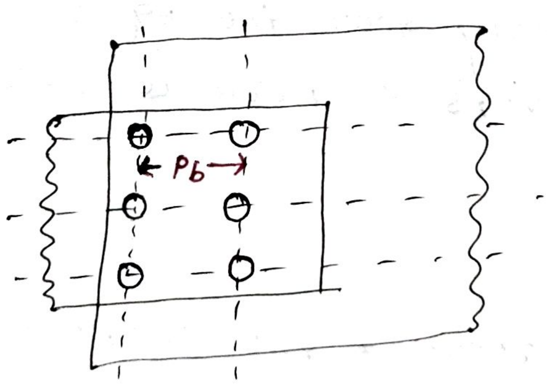
Important Terms used in Riveted Joints

(i) **Pitch** :- It is the distance from the centre of one rivet to the centre of the next rivet measured parallel to the seam. It is usually denoted by p.



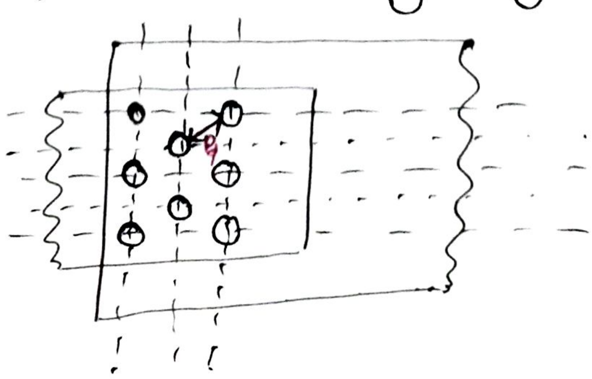
(ii) **Back pitch.**

→ It is perpendicular distance between the centre lines of the successive rows. It is usually denoted by P_b



(iii) **Diagonal pitch.**

→ It is the distance between the centre of the rivets in adjacent rows of zig-zag riveted joints

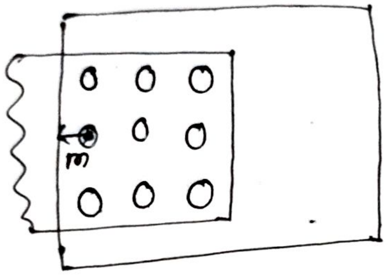


④ margin (m)

→ It is the distance between the centre of rivet hole to the nearest edge of the plate.

$$\boxed{m = 1.5d}$$

d = Diameter of rivet hole



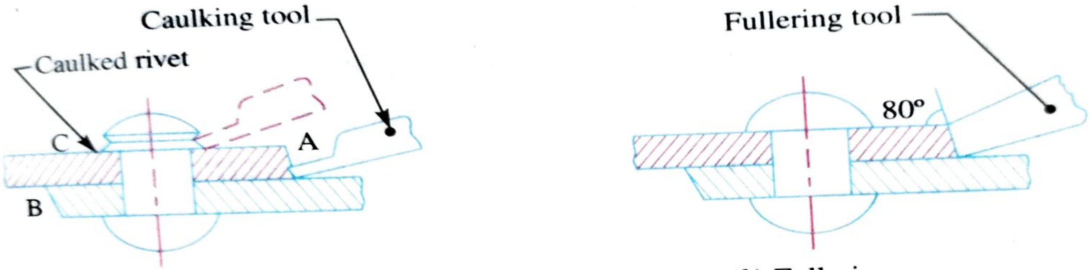
Caulking and Fullering

Caulking

→ It is used for closing down the edges of the plates and heads of the rivets to form a metal-to-metal joint

Fullering

→ It is used for pressing the surface of the edge of the plate. It helps to make fluid-tight joint



(*a*) Caulking.

(*b*) Fullering.

Fig. 9.12. Caulking and fullering.

Failure of a Riveted Joint

(i) Tearing failure
(ii) Shearing failure
(iii) Crushing failure

Tearing failure.

→ Due to Tensile stresses in the main plates the main plate or cover plates may be tear off across a row of rivets

→ The resistance offered by the plate against tearing is known as tearing resistance or tearing strength of the plate

p = pitch of the rivets
d = Diameter of the rivet hole
t = Thickness of the plate
σ_t = permissible Tensile stress

we know tearing Area

$$A_t = (p-d)\cdot t$$

tearing resistance

$$P_t = A_t \cdot \sigma_t = (p-d)t \cdot \sigma_t$$

$$\boxed{P_t = (p-d)t \cdot \sigma_t}$$

Shearing failure

→ The plate which are connected by the rivet exert tensile stress on the rivets, and if the rivet are unable to resist the stress, they are sheared off.

d = diameter of the rivet hole

τ = permissible shear stress

n = number of rivets

we know shearing area

$A_s = \frac{\pi}{4} \times d^2$ (in a single shear)

$= 2 \times \frac{\pi}{4} \times d^2$ (in double shear)

$= 1.875 \times \frac{\pi}{4} \times d^2$ (in double shear, according to Indian Boiler Regulation)

Shearing Resistance.

$P_s = n \times \frac{\pi}{4} \times d^2 \times \tau$ (in single shear)

$= n \times 2 \times \frac{\pi}{4} \times d^2 \times \tau$ (in double shear)

$= n \times 1.875 \times \frac{\pi}{4} \times d^2 \times \tau$ (in double shear, according to Indian Boiler Regulation)

(iii) Crushing failure.

→ Sometimes, the rivet do not actually shear off under the Tensile stress, but are crushed. Due to this, the rivet hole becomes of an oval shape and hence the joint become loose.

→ The resistance offered by a rivet to be crushed is known as crushing resistance.

d = diameter of the rivet hole

t = Thickness of the plate.

σ_C = permissible crushing stress

n = number of rivet

We know the crushing area

$$A_C = d \cdot t$$
$$= n \cdot d \cdot t$$

Crushing resistance (P_C)

$$\boxed{P_C = n \cdot d \cdot t \cdot \sigma_C}$$

Efficiency of riveted joint

→ It is the ratio between the strength of riveted joint and strength of un-riveted joint.

Strength of riveted joint
$$= \text{least of } P_t, P_S, P_C$$

Strength of un-riveted joint

$$P = p \times t \times \sigma_T$$

Efficiency of the riveted joint

$$\eta = \frac{\text{least of } P_t, P_s, P_c}{p \times t \times \sigma_T}$$

p = pitch of rivet

t = Thickness of the plate

σ_T = permissible Tensile stress

Find the efficiency of the following riveted joint

① Single riveted lap joint of 6mm plates with 20 mm diameter rivet having a pitch of 50 mm

② Double riveted lap joint of 6 mm plates with 20 mm diameter rivet having a pitch of 65 mm

Assume

permissible Tensile stress in plate = 120 mpa

permissible Shearing stress in rivet = 90 mpa

permissible crushing stress in rivet = 180 mpa

Solution

Given data

$t = 6\,mm$, $d = 20\,mm$, $\sigma_t = 120\,mpa = 120\,N/mm^2$

$\tau = 90\,mpa = 90\,N/mm^2$, $\sigma_c = 180\,mpa = 180\,N/mm^2$

pitch (p) = 50 mm

Tearing resistance of the plate

$P_t = (p-d)t \times \sigma_t = (50-20)\,6 \times 120 = 21600N$

(ii) Shearing resistance of the rivet

$$P_s = \frac{\pi}{4} \times d^2 \times \tau = \frac{\pi}{4} \times (20)^2 \times 90 = 28278\ N$$

(iii) Crushing resistance of the rivet

$$P_c = d \times t \times \sigma_c = 20 \times 6 \times 180 = 21600 N$$

Strength of the joint

$$= \text{least of } P_t, P_s, P_c = 21600 N$$

Strength of the unriveted joint

$$P = p \times t \times \sigma_t = 50 \times 6 \times 120 = 36{,}000\ N$$

Efficiency of the joint

$$\eta = \frac{\text{Least of } P_t, P_s \text{ and } P_c}{P} = \frac{21600}{36000} = 0.60 \text{ OR } 60\ \%.$$

(2) pitch $p = 65$ mm

(i) Tearing resistance of the plate

$$P_t = (p-d)\, t \times \sigma_t = (65-20)\, 6 \times 120 = 324000 N$$

(ii) Shearing resistance of the rivets

$$P_s = n \times \frac{\pi}{4} \times d^2 \times \tau = 2 \times \frac{\pi}{4} \times (20)^2 \times 90$$

$$= 56556\ N$$

(iii) Crushing resistance of the rivets

$$P_c = n \times d \times t \times \sigma_c = 2 \times 20 \times 6 \times 180 = 43200\ N$$

$$\text{Efficiency} = \frac{\text{least of } P_t, P_s, P_c}{(P) \text{ strength of unriveted joint}}$$

$$= \frac{324000}{46800} = 0.69 = 69\ \%$$

Example 9.3. *A double riveted double cover butt joint in plates 20 mm thick is made with 25 mm diameter rivets at 100 mm pitch. The permissible stresses are :*

$$\sigma_t = 120 \text{ MPa}; \qquad \tau = 100 \text{ MPa}; \qquad \sigma_c = 150 \text{ MPa}$$

Find the efficiency of joint, taking the strength of the rivet in double shear as twice than that of single shear.

Solution. Given : $t = 20$ mm ; $d = 25$ mm ; $p = 100$ mm ; $\sigma_t = 120$ MPa $= 120$ N/mm^2 ; $\tau = 100$ MPa $= 100$ N/mm^2 ; $\sigma_c = 150$ MPa $= 150$ N/mm^2

First of all, let us find the tearing resistance of the plate, shearing resistance and crushing resistance of the rivet.

(i) Tearing resistance of the plate

We know that tearing resistance of the plate per pitch length,

$$P_t = (p - d)\, t \times \sigma_t = (100 - 25)\, 20 \times 120 = 180\,000 \text{ N}$$

(ii) Shearing resistance of the rivets

Since the joint is double riveted butt joint, therefore the strength of two rivets in double shear is taken. We know that shearing resistance of the rivets,

$$P_s = n \times 2 \times \frac{\pi}{4} \times d^2 \times \tau = 2 \times 2 \times \frac{\pi}{4} (25)^2\, 100 = 196\,375 \text{ N}$$

(iii) Crushing resistance of the rivets

Since the joint is double riveted, therefore the strength of two rivets is taken. We know that crushing resistance of the rivets,

$$P_c = n \times d \times t \times \sigma_c = 2 \times 25 \times 20 \times 150 = 150\,000 \text{ N}$$

$\therefore$ Strength of the joint

$$= \text{Least of } P_t, P_s \text{ and } P_c$$
$$= 150\,000 \text{ N}$$

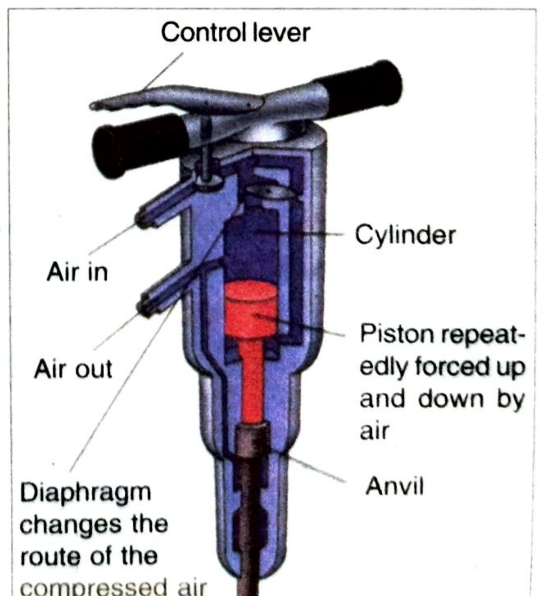
Efficiency of the joint

We know that the strength of the unriveted or solid plate,

$$P = p \times t \times \sigma_t = 100 \times 20 \times 120$$
$$= 240\,000 \text{ N}$$

$\therefore$ Efficiency of the joint

$$= \frac{\text{Least of } P_t, P_s \text{ and } P_c}{P} = \frac{150\,000}{240\,000}$$
$$= 0.625 \text{ or } 62.5\% \; \textbf{Ans.}$$



9.15 Design of Boiler Joints

Design of Boiler Joints

- The boiler has a longitudinal joint as well as circumferential joint
- longitudinal joint is used to get the required diameter of a boiler
- circumferential joint is used to get the required length of the boiler.

Design of longitudinal butt joint for a boiler

① Thickness of boiler shell (t)

$$t = \frac{p \cdot D}{2\sigma_t \times \eta_L} + 1\,mm$$

t = Thickness of the boiler shell
p = steam pressure in boiler
D = internal diameter of the boiler shell
σ_t = permissible tensile stress
η_L = Efficiency of the longitudinal joint

② Diameter of Rivets (d)

Unwin's formula

$$\boxed{d = 6\sqrt{t}}$$ (when t is greater than 8 mm)

- But if the thickness of plate is less than 8 mm then diameter of rivet hole may be calculated by equating shearing resistance and crushing resistance

$$\boxed{P_S = P_C}$$

(3) Pitch of Rivets

→ The pitch of the rivets is obtained by equating the tearing resistance of plate to the shearing resistance of the rivets.

It may be noted that

(a) The pitch of the rivets should not be less than 2d, which is necessary for the formation of head.

(b) The maximum value of the pitch of rivets for a longitudinal joint of a boiler as per IBR

$$P_{max} = C \times t + 41.28 \text{ mm}$$

t = Thickness of the shell plate

C = constant

Note

→ If the pitch of rivet as obtained by equating the tearing resistance to shearing resistance is more than P_{max} then the value of P_{max} is taken

(4) Distance between the rows of rivets (P_b)

(a) For equal number of rivets in more than one row for lap joint or butt joint

$$P_b = 0.33P + 0.67d, \text{ for zig-zag riveting}$$
$$= 2d, \text{ for chain riveting}$$

(b) For joints in which the number of rivets in outer rows is half the number of rivets in inner rows and if the inner rows are chain riveted

$\boxed{P_b = 0.33P + 0.67 \text{ or } 2d}$, whichever is greater

(c) For joints in which the number of rivets in outer rows is half the number of rivets in inner rows and if inner rows are zig - zag riveted

$$\boxed{P_b = 0.2P + 1.15d}$$

→ The distance between the rows in which there are full number of rivets (zig-zag)

$$\boxed{P_b = 0.165P + 0.67d}$$

(5) Thickness of butt strap

(a) The thickness of butt strap, in no case, shall be less than 10 mm

(b) $t_1 = 1.125t$, for chain riveting, single butt strap

$t_1 = 1.125t\left(\frac{P-d}{P-2d}\right)$, for single butt strap, every alternate rivet in outer rows being omitted

$t_1 = 0.625t$, for double strap (chain riveting), for equal width

$t_1 = 0.625t\left(\frac{P-d}{P-2d}\right)$, for double strap, every ~~equal~~ alternate rivet in the outer rows being omitted, for equal width

(c) For unequal width

$t_1 = 0.75t$, for wide strap on the inside

$t_2 = 0.625t$, for narrow strap on the outside.

(6) Margin (m) $\boxed{m = 1.5d}$

Design of circumferential lap joint for a boiler

① Thickness of the shell and diameter of rivet –

→ The thickness of the boiler shell and the diameter of the rivet will be same as for longitudinal joint

② Number of rivets.

→ Since it is a lap joint, therefore the rivets will be in single shear

Shearing resistance of the rivet

$$\boxed{P_s = n \times \frac{\pi}{4} \times d^2 \times \tau} \qquad \text{(i)}$$

n = Total number of rivets

we know that the inner diameter of the boiler shell (D), and the pressure of the steam (P), the total shearing load acting on the circumferential joint

$$W_s = \frac{\pi}{4} \times D^2 \times P \qquad \text{(ii)}$$

- from equation (i) and (ii) we get

$$n \times \frac{\pi}{4} \times d^2 \times \tau = \frac{\pi}{4} \times D^2 \times P$$

$$\boxed{n = \left(\frac{D}{d}\right)^2 \times \frac{P}{\tau}}$$

③ pitch of rivets (P_1)

$$\eta_c = \frac{P_1 - d}{P_1}$$

(4) Number of Rows

$$= \frac{\text{Total number of Rivet}}{\text{number of Rivets in one Row}}$$

Number of Rivet in one Row

$$\boxed{= \frac{\pi(D+t)}{P_1}}$$

(5) margin $= \boxed{1.5d}$

Table 9.3. Size of rivet diameters for rivet hole diameter as per IS : 1928 – 1961 (Reaffirmed 1996).

Basic size of rivet mm	12	14	16	18	20	22	24	27	30	33	36	39	42	48
Rivet hole diameter (min) mm	13	15	17	19	21	23	25	28.5	31.5	34.5	37.5	41	44	50

According to IS : 1928 – 1961 (Reaffirmed 1996), the table on the next page (Table 9.4) gives

Preferred numbers are indicated by ×.

Table 9.5. Values of constant *C*.

Number of rivets per pitch length	*Lap joint*	*Butt joint (single strap)*	*Butt joint (double strap)*
1	1.31	1.53	1.75
2	2.62	3.06	3.50
3	3.47	4.05	4.63
4	4.17	–	5.52
5	–	–	6.00

Example 9.4. *A double riveted lap joint with zig-zag riveting is to be designed for 13 mm thick plates. Assume*

$$\sigma_t = 80 \text{ MPa} \; ; \; \tau = 60 \text{ MPa} \; ; \text{ and } \sigma_c = 120 \text{ MPa}$$

State how the joint will fail and find the efficiency of the joint.

Solution. Given : $t = 13$ mm ; $\sigma_t = 80$ MPa $= 80$ N/mm^2 ; $\tau = 60$ MPa $= 60$ N/mm^2 ; $\sigma_c = 120$ MPa $= 120$ N/mm^2

1. *Diameter of rivet*

Since the thickness of plate is greater than 8 mm, therefore diameter of rivet hole,

$$d = 6\sqrt{t} = 6\sqrt{13} = 21.6 \text{ mm}$$

From Table 9.3, we find that according to IS : 1928 – 1961 (Reaffirmed 1996), the standard size of the rivet hole (d) is 23 mm and the corresponding diameter of the rivet is 22 mm. **Ans.**

2. Pitch of rivets

Let p = Pitch of the rivets.

Since the joint is a double riveted lap joint with zig-zag riveting [See Fig. 9.6 (*c*)], therefore there are two rivets per pitch length, *i.e.* $n = 2$. Also, in a lap joint, the rivets are in single shear.

We know that tearing resistance of the plate,

$$P_t = (p - d)t \times \sigma_t = (p - 23)\ 13 \times 80 = (p - 23)\ 1040 \text{ N} \quad ...(i)$$

and shearing resistance of the rivets,

$$P_s = n \times \frac{\pi}{4} \times d^2 \times \tau = 2 \times \frac{\pi}{4}(23)^2\ 60 = 49\ 864 \text{ N} \quad ...(ii)$$

...($\because$ There are two rivets in single shear)

From equations (*i*) and (*ii*), we get

$$p - 23 = 49864 / 1040 = 48 \quad \text{or} \quad p = 48 + 23 = 71 \text{ mm}$$

The maximum pitch is given by,

$$p_{max} = C \times t + 41.28 \text{ mm}$$

From Table 9.5, we find that for 2 rivets per pitch length, the value of C is 2.62.

$$\therefore \quad p_{max} = 2.62 \times 13 + 41.28 = 75.28 \text{ mm}$$

Since p_{max} is more than p, therefore we shall adopt

$$p = 71 \text{ mm} \quad \textbf{Ans.}$$

3. Distance between the rows of rivets

We know that the distance between the rows of rivets (for zig-zag riveting),

$$p_b = 0.33\,p + 0.67\,d = 0.33 \times 71 + 0.67 \times 23 \text{ mm}$$
$$= 38.8 \text{ say } 40 \text{ mm} \quad \textbf{Ans.}$$

4. Margin

We know that the margin,

$$m = 1.5\,d = 1.5 \times 23 = 34.5 \text{ say } 35 \text{ mm} \quad \textbf{Ans.}$$

Failure of the joint

Now let us find the tearing resistance of the plate, shearing resistance and crushing resistance of the rivets.

We know that tearing resistance of the plate,

$$P_t = (p - d)\ t \times \sigma_t = (71 - 23)13 \times 80 = 49\ 920 \text{ N}$$

Shearing resistance of the rivets,

$$P_s = n \times \frac{\pi}{4} \times d^2 \times \tau = 2 \times \frac{\pi}{4}(23)^2\ 60 = 49\ 864 \text{ N}$$

and crushing resistance of the rivets,

$$P_c = n \times d \times t \times \sigma_c = 2 \times 23 \times 13 \times 120 = 71\ 760 \text{ N}$$

The least of P_t, P_s and P_c is $P_s = 49\ 864$ N. Hence the joint will fail due to shearing of the rivets. **Ans.**

Efficiency of the joint

We know that strength of the unriveted or solid plate,

$$P = p \times t \times \sigma_t = 71 \times 13 \times 80 = 73\ 840 \text{ N}$$

$\therefore$ Efficiency of the joint,

$$\eta = \frac{P_s}{P} = \frac{49\ 864}{73\ 840} = 0.675 \text{ or } 67.5\% \quad \textbf{Ans.}$$

fail due to tearing off the plate.

Example 9.6. *Two plates of 10 mm thickness each are to be joined by means of a single riveted double strap butt joint. Determine the rivet diameter, rivet pitch, strap thickness and efficiency of the joint. Take the working stresses in tension and shearing as 80 MPa and 60 MPa respectively.*

Solution. Given : $t = 10$ mm ; $\sigma_t = 80$ MPa $= 80$ N/mm^2 ; $\tau = 60$ MPa $= 60$ N/mm^2

1. Diameter of rivet

Since the thickness of plate is greater than 8 mm, therefore diameter of rivet hole,

$$d = 6\sqrt{t} = 6\sqrt{10} = 18.97 \text{ mm}$$

From Table 9.3, we see that according to IS : 1928 – 1961 (Reaffirmed 1996), the standard diameter of rivet hole (d) is 19 mm and the corresponding diameter of the rivet is 18 mm. **Ans.**

2. Pitch of rivets

Let p = Pitch of rivets.

Since the joint is a single riveted double strap butt joint as shown in Fig. 9.8, therefore there is one rivet per pitch length (*i.e.* $n = 1$) and the rivets are in double shear.

We know that tearing resistance of the plate,

$$P_t = (p - d)\,t \times \sigma_t = (p - 19)10 \times 80 = 800\,(p - 19) \text{ N} \qquad ...(i)$$

and shearing resistance of the rivets,

$$P_s = n \times 1.875 \times \frac{\pi}{4} \times d^2 \times \tau \qquad ...(\because \text{ Rivets are in double shear})$$

$$= 1 \times 1.875 \times \frac{\pi}{4}(19)^2\,60 = 31\,900 \text{ N} \qquad ...(\because n = 1) \quad ...(ii)$$

From equations (*i*) and (*ii*), we get

$$800\,(p - 19) = 31\,900$$

$$\therefore \quad p - 19 = 31\,900 / 800 = 39.87 \text{ or } p = 39.87 + 19 = 58.87 \text{ say } 60 \text{ mm}$$

According to I.B.R., the maximum pitch of rivets,

$$p_{max} = C.t + 41.28 \text{ mm}$$

From Table 9.5, we find that for double strap butt joint and 1 rivet per pitch length, the value of C is 1.75.

$$\therefore \quad p_{max} = 1.75 \times 10 + 41.28 = 58.78 \text{ say } 60 \text{ mm}$$

From above we see that $p = p_{max} = 60$ mm **Ans.**

3. Thickness of cover plates

We know that thickness of cover plates,

$$t_1 = 0.625\, t = 0.625 \times 10 = 6.25 \text{ mm} \quad \textbf{Ans.}$$

Efficiency of the joint

We know that tearing resistance of the plate,

$$P_t = (p - d)\, t \times \sigma_t = (60 - 19)\, 10 \times 80 = 32\ 800 \text{ N}$$

and shearing resistance of the rivets,

$$P_s = n \times 1.875 \times \frac{\pi}{4} \times d^2 \times \tau = 1 \times 1.875 \times \frac{\pi}{4}\, (19)^2\, 60 = 31\ 900 \text{ N}$$

$\therefore$ Strength of the joint

$$= \text{Least of } P_t \text{ and } P_s = 31\ 900 \text{ N}$$

Strength of the unriveted plate per pitch length

$$P = p \times t \times \sigma_t = 60 \times 10 \times 80 = 48\ 000 \text{ N}$$

$\therefore$ Efficiency of the joint,

$$\eta = \frac{\text{Least of } P_t \text{ and } P_s}{P} = \frac{31\ 900}{48\ 000} = 0.665 \text{ or } 66.5\% \quad \textbf{Ans.}$$

Example 9.7. *Design a double riveted butt joint with two cover plates for the longitudinal seam of a boiler shell 1.5 m in diameter subjected to a steam pressure of 0.95 N/mm². Assume joint efficiency as 75%, allowable tensile stress in the plate 90 MPa ; compressive stress 140 MPa ; and shear stress in the rivet 56 MPa.*

Solution. Given : $D = 1.5$ m $= 1500$ mm ; $P = 0.95$ N/mm² ; $\eta_l = 75\% = 0.75$; $\sigma_t = 90$ MPa $= 90$ N/mm² ; $\sigma_c = 140$ MPa $= 140$ N/mm² ; $\tau = 56$ MPa $= 56$ N/mm²

1. Thickness of boiler shell plate

We know that thickness of boiler shell plate,

$$t = \frac{P.D}{2\sigma_t \times \eta_l} + 1 \text{ mm} = \frac{0.95 \times 1500}{2 \times 90 \times 0.75} + 1 = 11.6 \text{ say } 12 \text{ mm} \quad \textbf{Ans.}$$

2. Diameter of rivet

Since the thickness of the plate is greater than 8 mm, therefore the diameter of the rivet hole,

$$d = 6\sqrt{t} = 6\sqrt{12} = 20.8 \text{ mm}$$

From Table 9.3, we see that according to IS : 1928 – 1961 (Reaffirmed 1996), the standard diameter of the rivet hole (d) is 21 mm and the corresponding diameter of the rivet is 20 mm. **Ans.**

3. Pitch of rivets

Let $\quad p$ = Pitch of rivets.

The pitch of the rivets is obtained by equating the tearing resistance of the plate to the shearing resistance of the rivets.

We know that tearing resistance of the plate,

$$P_t = (p - d)\, t \times \sigma_t = (p - 21)12 \times 90 = 1080\, (p - 21) \text{N} \qquad \ldots(i)$$

Since the joint is double riveted double strap butt joint, as shown in Fig. 9.9, therefore there are two rivets per pitch length (*i.e.* $n = 2$) and the rivets are in double shear. Assuming that the rivets in double shear are 1.875 times stronger than in single shear, we have

Shearing strength of the rivets,

$$P_s = n \times 1.875 \times \frac{\pi}{4} \times d^2 \times \tau = 2 \times 1.875 \times \frac{\pi}{4} (21)^2 \times 56 \text{ N} \qquad ...(ii)$$

$$= 72\ 745 \text{ N}$$

From equations (i) and (ii), we get

$$1080\,(p - 21) = 72\ 745$$

$\therefore$ $p - 21 = 72\ 745 / 1080 = 67.35$ or $p = 67.35 + 21 = 88.35$ say 90 mm

According to I.B.R., the maximum pitch of rivets for longitudinal joint of a boiler is given by

$$p_{max} = C \times t + 41.28 \text{ mm}$$

From Table 9.5, we find that for a double riveted double strap butt joint and two rivets per pitch length, the value of C is 3.50.

$\therefore$ $p_{max} = 3.5 \times 12 + 41.28 = 83.28$ say 84 mm

Since the value of p is more than p_{max}, therefore we shall adopt pitch of the rivets,

$$p = p_{max} = 84 \text{ mm} \quad \textbf{Ans.}$$

4. *Distance between rows of rivets*

Assuming zig-zag riveting, the distance between the rows of the rivets (according to I.B.R.),

$$p_b = 0.33\,p + 0.67\,d = 0.33 \times 84 + 0.67 \times 21 = 41.8 \text{ say } 42 \text{ mm} \quad \textbf{Ans.}$$

5. *Thickness of cover plates*

According to I.B.R., the thickness of each cover plate of equal width is

$$t_1 = 0.625\,t = 0.625 \times 12 = 7.5 \text{ mm} \quad \textbf{Ans.}$$

6. *Margin*

We know that the margin,

$$m = 1.5\,d = 1.5 \times 21 = 31.5 \text{ say } 32 \text{ mm} \quad \textbf{Ans.}$$

Let us now find the efficiency for the designed joint.

Tearing resistance of the plate,

$$P_t = (p - d)\,t \times \sigma_t = (84 - 21)12 \times 90 = 68\ 040 \text{ N}$$

Shearing resistance of the rivets,

$$P_s = n \times 1.875 \times \frac{\pi}{4} \times d^2 \times \tau = 2 \times 1.875 \times \frac{\pi}{4} (21)^2 \times 56 = 72\ 745 \text{ N}$$

and crushing resistance of the rivets,

$$P_c = n \times d \times t \times \sigma_c = 2 \times 21 \times 12 \times 140 = 70\ 560 \text{ N}$$

Since the strength of riveted joint is the least value of P_t, P_s or P_c, therefore strength of the riveted joint,

$$P_t = 68\ 040 \text{ N}$$

We know that strength of the un-riveted plate,

$$P = p \times t \times \sigma_t = 84 \times 12 \times 90 = 90\ 720 \text{ N}$$

$\therefore$ Efficiency of the designed joint,

$$\eta = \frac{P_t}{P} = \frac{68\ 040}{90\ 720} = 0.75 \text{ or } 75\% \quad \textbf{Ans.}$$

Since the efficiency of the designed joint is equal to the given efficiency of 75%, therefore the design is satisfactory.

Welding Joint

→ A welding is a permanent joint. in which two or more of similar and dissimilar material can be joined together by heat with or without application of pressure and a filler material

Advantages and Disadvantages of welding joint over Riveted joint

Advantages.

1. The welded structures are usually lighter than Riveted structures
2. The welded joints provide maximum efficiency
3. Alternation and Additions can be easily made in the existing structures.
4. welded structure is smooth in appearance.
5. A welded joint has a greater strength.
6. The welding provides very Rigid joints.
7. The process of welding takes less time than the Riveting.

Disadvantages.

1. Since There are uneven heating and cooling during fabrication
2. It requires a highly skilled labour.
3. No provision is kept for expansion and contraction in the frame, Therefore There is a possibility of cracks developing in it
4. The inspection of welding work is more difficult than Riveting work

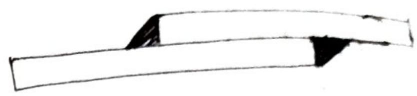
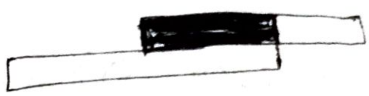
Types of welded joint

① Lap joint or fillet joint
② Butt joint

Lap joint

→ The lap joint or the fillet joint is obtained by overlapping the plates and then welded the edges of the plates.

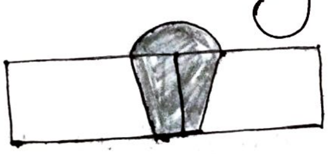
The fillet joint may be

① Single Transverse fillet 
② Double Transverse fillet 
③ Parallel fillet joints 

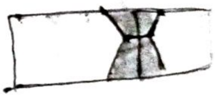
Butt joint

→ The butt joint is obtaining by placing the plates edge ~~to edge and~~ then welding together.

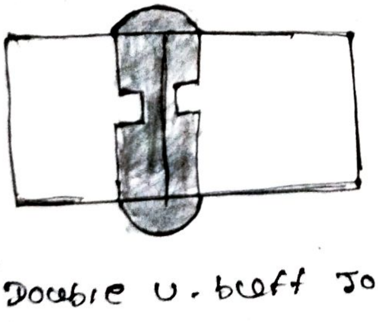

(Square butt joint)


(Single V-butt joint)


(Single U-butt joint)


(Double V-butt joint)

Butt joint

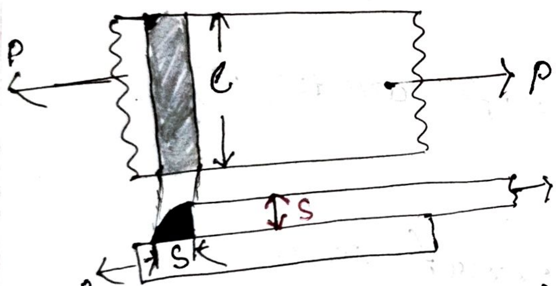

(Double U-butt joint)

The butt joint may be

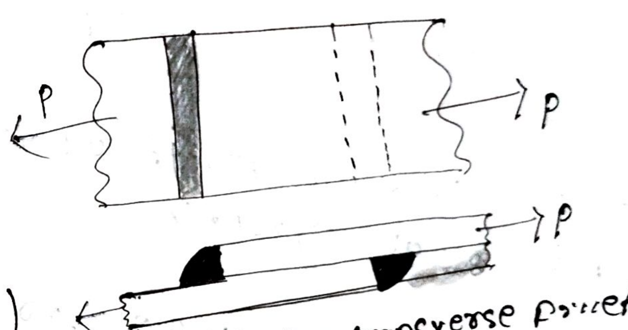
1. Square butt joint
2. Single V-butt joint
3. Single U-butt joint
4. Double V-butt joint
5. Double U-butt joint

Strength of Transverse fillet welded joint

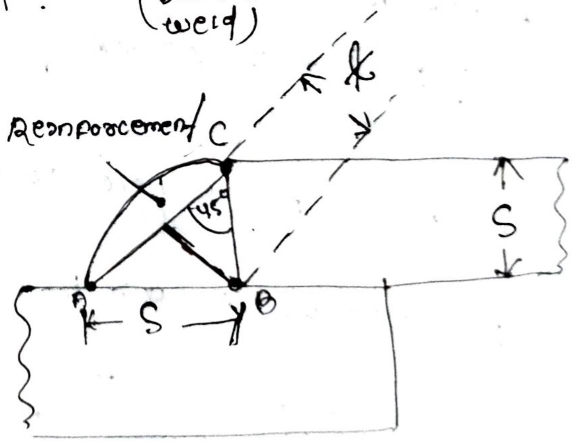
→ The transverse fillet welds are designed for tensile strength



(Single transverse fillet weld)



(Double transverse fillet weld)



t = Throat thickness

s = leg or size of weld
= Thickness of plate

l = length of weld

The throat thickness

$$t = s \times \sin 45 = 0.707s$$

Area of the weld or Throat Area

$$A = \text{Throat thickness} \times \text{length of weld}$$

$$= t \times l = 0.707s \times l$$

Strength for single fillet weld

P = Throat Area x Allowable Tensile Stress

$$\boxed{P = 0.707S \times l \times \sigma_T}$$

Strength for double fillet weld

$P = 2 \times 0.707S \times l \times \sigma_T$

$$\boxed{P = 1.414S \times l \times \sigma_T}$$

Strength of parallel fillet welded joint

→ The parallel fillet welded joints are designed for shear strength.

minimum Area of weld or The Throat Area

$$\boxed{A = 0.707S \times l}$$

Strength for single parallel fillet weld

P = Throat Area x Allowable shear stress

$$\boxed{P = 0.707S \times l \times \tau}$$

Strength for double parallel fillet weld

$P = 2 \times 0.707S \times l \times \tau = \boxed{1.414S \times l \times \tau}$

Example 10.1. *A plate 100 mm wide and 10 mm thick is to be welded to another plate by means of double parallel fillets. The plates are subjected to a static load of 80 kN. Find the length of weld if the permissible shear stress in the weld does not exceed 55 MPa.*

Electric arc welding

Solution. Given: *Width = 100 mm ; Thickness = 10 mm ; $P = 80$ kN $= 80 \times 10^3$ N ; $\tau = 55$ MPa $= 55$ N/mm^2

Let l = Length of weld, and

s = Size of weld = Plate thickness = 10 mm ... (Given)

We know that maximum load which the plates can carry for double parallel fillet weld (P),

$$80 \times 10^3 = 1.414 \times s \times l \times \tau = 1.414 \times 10 \times l \times 55 = 778\ l$$

$$\therefore \quad l = 80 \times 10^3 / 778 = 103 \text{ mm}$$

Adding 12.5 mm for starting and stopping of weld run, we have

$$l = 103 + 12.5 = 115.5 \text{ mm} \quad \textbf{Ans.}$$

* Superfluous data.

Special cases of fillet welded joints

① **Circular fillet weld subjected to torsion –**

d = Diameter of Rod

r = Radius of Rod

T = Torque Acting on the Rod

S = Size of weld

t = Throat Thickness

J = Polar moment of inertia of the weld section $= \frac{\pi t d^3}{4}$

Shear stress for the material $(\tau) = \frac{2T}{\pi t d^2}$

maximum shear stress $(\tau_{max}) = \frac{2.83T}{\pi S d^2}$

② Circular fillet weld subjected to bending moment

d = Diameter of Rod

M = Bending moment.

S = Size of weld

t = Throat Thickness

Z = section modulus of the weld section

$= \frac{\pi t d^2}{4}$

Bending stress $\sigma_b = \frac{M}{Z} = \frac{4M}{\pi t d^2}$

maximum Bending stress

$$(\sigma_b)_{max} = \frac{5.66M}{\pi S d^2}$$

③ Long fillet weld subjected to Torsion

T = Torque acting on the vertical plate

l = length of weld

S = size of weld

t = Throat Thickness

J = polar moment of inertia of weld section.

Shear stress $(\tau) = \frac{3T}{t \times l^2}$

maximum Shear stress $\tau_{max} = \frac{4.242T}{S \times l^2}$

Example 10.2. *A 50 mm diameter solid shaft is welded to a flat plate by 10 mm fillet weld as shown in Fig. 10.12. Find the maximum torque that the welded joint can sustain if the maximum shear stress intensity in the weld material is not to exceed 80 MPa.*

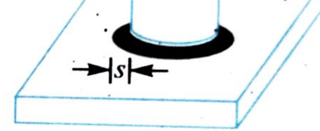


Fig. 10.12

Solution. Given : $d = 50$ mm ; $s = 10$ mm ; $\tau_{max} = 80$ MPa $= 80$ N/mm^2

Let T = Maximum torque that the welded joint can sustain.

We know that the maximum shear stress (τ_{max}),

$$80 = \frac{2.83\,T}{\pi s \times d^2} = \frac{2.83T}{\pi \times 10\,(50)^2} = \frac{2.83\,T}{78550}$$

$$\therefore \quad T = 80 \times 78\,550/2.83$$

$$= 2.22 \times 10^6 \text{ N-mm} = 2.22 \text{ kN-m} \quad \textbf{Ans.}$$

Example 10.3. *A plate 1 m long, 60 mm thick is welded to another plate at right angles to each other by 15 mm fillet weld, as shown in Fig. 10.13. Find the maximum torque that the welded joint can sustain if the permissible shear stress intensity in the weld material is not to exceed 80 MPa.*

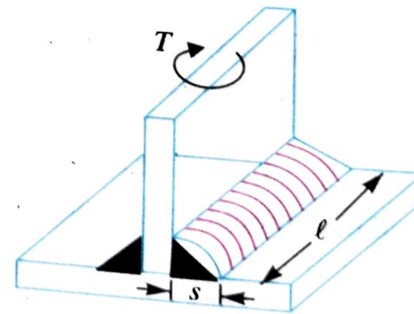


Fig. 10.13

Solution. Given: $l = 1$m $= 1000$ mm ; Thickness $= 60$ mm ; $s = 15$ mm ; $\tau_{max} = 80$ MPa $= 80$ N/mm^2

Let T = Maximum torque that the welded joint can sustain.

We know that the maximum shear stress (τ_{max}),

$$80 = \frac{4.242\,T}{s \times l^2} = \frac{4.242\,T}{15\,(1000)^2} = \frac{0.283\,T}{10^6}$$

$$\therefore \quad T = 80 \times 10^6 / 0.283 = 283 \times 10^6 \text{ N-mm} = 283 \text{ kN-m} \quad \textbf{Ans.}$$

10.19 Strength of Butt Joints

Design of coupling

Shaft coupling

→ Shafts are usually available up to 7meter length due to inconvenience in transport. in order to have a greter length, it becomes necessary to joint two or more pieces of the shaft by means of a coupling

→ It is an arrangement by which two shaft can be joint together through temporary joint. This temporary joint is known as coupling.

Requirements of a good shaft coupling

A good shaft coupling should have following requirement

1. It should be easy to connect or disconnect
2. It should transmit the full power from one shaft to the other shaft without losses
3. It should hold the shafts in perfect alignment
4. It should be reduce the transmission of shock load from one shaft to another shaft
5. It should be no projecting parts.

Types of Shaft coupling

1. Rigid coupling :- It is used to connect two shaft which are perfectly aligned.

Following types of Rigid couplings are
- (a) Sleeve or muff coupling
- (b) Clamp or split-muff or compression coupling
- (c) Flange coupling.

(3) Flexible coupling

→ It is used to connect two shaft having both lateral and angular misalignment.

Flexible coupling are following types.

(a) Bushed pin type coupling
(b) Universal coupling
(c) Oldham coupling

Sleeve or muff coupling

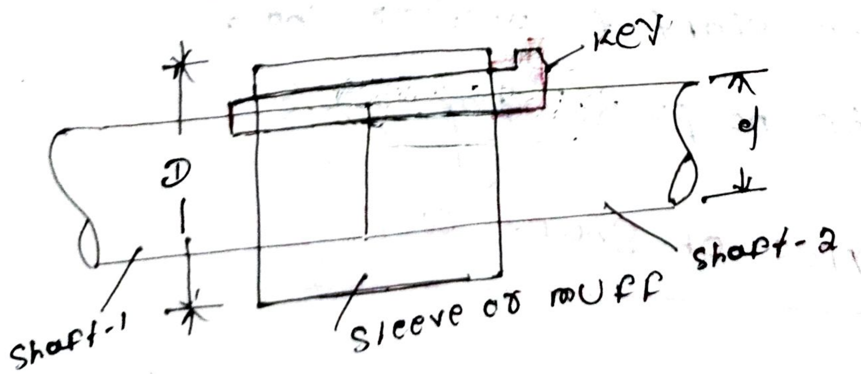
→ It is the simplest type of rigid coupling, made of cast iron

→ outer diameter of the sleeve $D = 2d + 13$ mm

length of sleeve $L = 3.5d$

d = diameter of the shaft

(1) Design for sleeve



T = Torque to be transmitted by the coupling

τ_c = permissible shear stress

Torque transmitted by Hollow section

$$T = \frac{\pi}{16} \times \tau_c \left[\frac{D^4 - d^4}{D}\right] = \frac{\pi}{16} \times \tau_c \times D^3 (1 - K^4)$$

$\therefore K = \frac{d}{D}$

② **Design for key**

length of the key (l)

$$l = \frac{L}{2} = \frac{3.5d}{2}.$$

Torque transmitted

$$T = l \times w \times \tau \times \frac{d}{2}$$ (considering shearing of the key)

$$T = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2}$$ (considering crushing of the key)

the same end for each shaft or they may be driven from opposite ends.

Example 13.4. *Design and make a neat dimensioned sketch of a muff coupling which is used to connect two steel shafts transmitting 40 kW at 350 r.p.m. The material for the shafts and key is plain carbon steel for which allowable shear and crushing stresses may be taken as 40 MPa and 80 MPa respectively. The material for the muff is cast iron for which the allowable shear stress may be assumed as 15 MPa.*

Solution. Given : P = 40 kW = 40 × 10³ W; N = 350 r.p.m.; τ_s = 40 MPa = 40 N/mm²; σ_{cs} = 80 MPa = 80 N/mm²; τ_c = 15 MPa = 15 N/mm²

A type of muff couplings.

Note : This picture is given as additional information and is not a direct example of the current chapter.

The muff coupling is shown in Fig. 13.10. It is designed as discussed below :

1. Design for shaft

Let d = Diameter of the shaft.

We know that the torque transmitted by the shaft, key and muff,

$$T = \frac{P \times 60}{2\,\pi N} = \frac{40 \times 10^3 \times 60}{2\,\pi \times 350} = 1100 \text{ N-m}$$

$$= 1100 \times 10^3 \text{ N-mm}$$

We also know that the torque transmitted (T),

$$1100 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$$\therefore \quad d^3 = 1100 \times 10^3 / 7.86 = 140 \times 10^3 \text{ or } d = 52 \text{ say } 55 \text{ mm}$$ **Ans.**

2. Design for sleeve

We know that outer diameter of the muff,

$$D = 2d + 13 \text{ mm} = 2 \times 55 + 13 = 123 \text{ say } 125 \text{ mm}$$ **Ans.**

and length of the muff,

$$L = 3.5\, d = 3.5 \times 55 = 192.5 \text{ say } 195 \text{ mm}$$ **Ans.**

Let us now check the induced shear stress in the muff. Let τ_c be the induced shear stress in the muff which is made of cast iron. Since the muff is considered to be a hollow shaft, therefore the torque transmitted (T),

$$1100 \times 10^3 = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D}\right) = \frac{\pi}{16} \times \tau_c \left[\frac{(125)^4 - (55)^4}{125}\right]$$

$$= 370 \times 10^3\, \tau_c$$

$$\therefore \quad \tau_c = 1100 \times 10^3 / 370 \times 10^3 = 2.97 \text{ N/mm}^2$$

Since the induced shear stress in the muff (cast iron) is less than the permissible shear stress of 15 N/mm², therefore the design of muff is safe.

3. Design for key

From Table 13.1, we find that for a shaft of 55 mm diameter,

Width of key, w = 18 mm **Ans.**

Since the crushing stress for the key material is twice the shearing stress, therefore a square key may be used.

∴ Thickness of key, $t = w = 18$ mm **Ans.**

We know that length of key in each shaft,

$$l = L / 2 = 195 / 2 = 97.5 \text{ mm} \quad \textbf{Ans.}$$

Let us now check the induced shear and crushing stresses in the key. First of all, let us consider shearing of the key. We know that torque transmitted (T),

$$1100 \times 10^3 = l \times w \times \tau_s \times \frac{d}{2} = 97.5 \times 18 \times \tau_s \times \frac{55}{2} = 48.2 \times 10^3 \, \tau_s$$

$$\therefore \quad \tau_s = 1100 \times 10^3 / 48.2 \times 10^3 = 22.8 \text{ N/mm}^2$$

Now considering crushing of the key. We know that torque transmitted (T),

$$1100 \times 10^3 = l \times \frac{t}{2} \times \sigma_{cs} \times \frac{d}{2} = 97.5 \times \frac{18}{2} \times \sigma_{cs} \times \frac{55}{2} = 24.1 \times 10^3 \, \sigma_{cs}$$

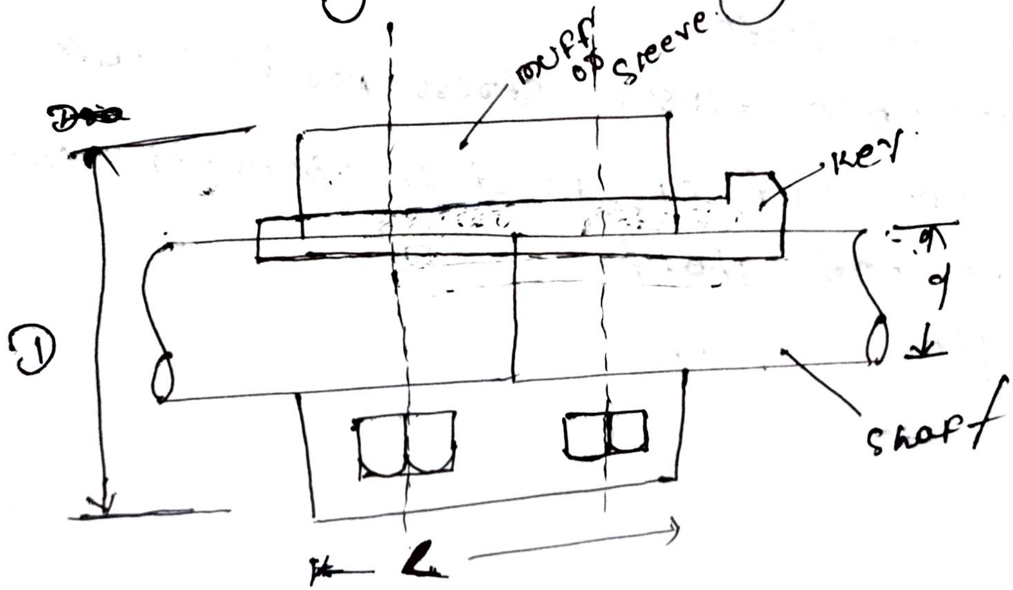
$$\therefore \quad \sigma_{cs} = 1100 \times 10^3 / 24.1 \times 10^3 = 45.6 \text{ N/mm}^2$$

Since the induced shear and crushing stresses are less than the permissible stresses, therefore the design of key is safe.

Clamp or compression coupling

→ It is also known as split muff coupling. In this case, The muff or sleeve is made into two halves and are bolted together.

→ The Advantage of this coupling is that the position of the shafts need not be changed for assembling or disassembling of the coupling.



Diameter of the muff or sleeve

$$\boxed{D = 2d + 13 \text{ mm}}$$

length of the muff or sleeve

$$\boxed{L = 3.5d}$$

① Design of muff and key

Sleeve

$$T = \frac{\pi}{16} \times \tau_c \times \left[\frac{D^4 - d^4}{D}\right] = \frac{\pi}{16} \times \tau_c \times D^3 (1 - K^4)$$

$$\boxed{\because K = \frac{d}{D}}$$

Key

$$l = \frac{L}{2} = \frac{3.5d}{2}$$

$$T = l \times w \times \tau \times \frac{d}{2} \text{ (considering shearing of the key)}$$

$$= l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \text{ (considering crushing of the key)}$$

② Design of clamping bolts

T = Torque transmitted by the shaft

d = Diameter of shaft

d_b = Root or effective diameter of bolt

n = Number of bolts

σ_t = permissible tensile stress for bolt material

μ = co-efficient of friction between the muff and shaft

L = length of muff

Force exerted by each bolt

$$= \frac{\pi}{4} \times d_b^2 \times \sigma_t$$

Force exerted by the bolt on each side of the shaft

$$= \frac{\pi}{4} \times d_b^2 \times \sigma_t \times \frac{n}{2}.$$

Let p be the pressure on the shaft and the muff surface due to the force, then for the uniform pressure distribution over the surface

$$p = \frac{\text{Force}}{\text{Projected Area}} = \frac{\frac{\pi}{4} \times d_b^2 \times \sigma_t \times \frac{n}{2}}{\frac{1}{2} \times L \times d}$$

Frictional force between each shaft and muff

$$F = \mu \times \text{pressure} \times \text{Area} = \mu \times p \times \frac{1}{2} \times \pi d \times L$$

$$= \mu \times \frac{\frac{\pi}{4} \times d_b^2 \times \sigma_t \times \frac{n}{2}}{\frac{1}{2} \times L \times d} \times \frac{1}{2} \pi d \times L$$

$$= \mu \times \frac{\pi}{4} \times d_b^2 \times \sigma_t \times \frac{n}{2} \times \pi = \mu \times \frac{\pi^2}{8} \times d_b^2 \times \sigma_t \times n$$

Torque that can be transmitted by the coupling

$$T = F \times \frac{d}{2} = \mu \times \frac{\pi^2}{8} \times d_b^2 \times \sigma_t \times n \times \frac{d}{2} =$$

$$= \frac{\pi^2}{16} \times \mu \times d_b^2 \times \sigma_t \times n \times d$$

$$\boxed{\mu = 0.3}$$

Note: The value of μ may be taken as 0.3.

Example 13.5. *Design a clamp coupling to transmit 30 kW at 100 r.p.m. The allowable shear stress for the shaft and key is 40 MPa and the number of bolts connecting the two halves are six. The permissible tensile stress for the bolts is 70 MPa. The coefficient of friction between the muff and the shaft surface may be taken as 0.3.*

Solution. Given : $P = 30$ kW $= 30 \times 10^3$ W ; $N = 100$ r.p.m. ; $\tau = 40$ MPa $= 40$ N/mm^2 ; $n = 6$; $\sigma_t = 70$ MPa $= 70$ N/mm^2 ; $\mu = 0.3$

1. Design for shaft

Let d = Diameter of shaft.

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\,\pi N} = \frac{30 \times 10^3 \times 60}{2\,\pi \times 100} = 2865 \text{ N-m} = 2865 \times 10^3 \text{ N-mm}$$

We also know that the torque transmitted by the shaft (T),

$$2865 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$$\therefore \quad d^3 = 2865 \times 10^3 / 7.86 = 365 \times 10^3 \text{ or } d = 71.4 \text{ say } 75 \text{ mm } \textbf{Ans.}$$

2. Design for muff

We know that diameter of muff,

$$D = 2d + 13 \text{ mm} = 2 \times 75 + 13 = 163 \text{ say } 165 \text{ mm } \textbf{Ans.}$$

and total length of the muff,

$$L = 3.5\, d = 3.5 \times 75 = 262.5 \text{ mm } \textbf{Ans.}$$

3. Design for key

The width and thickness of the key for a shaft diameter of 75 mm (from Table 13.1) are as follows :

Width of key, $w = 22$ mm **Ans.**

Thickness of key, $t = 14$ mm **Ans.**

and length of key = Total length of muff = 262.5 mm **Ans.**

4. Design for bolts

Let d_b = Root or core diameter of bolt.

We know that the torque transmitted (T),

$$2865 \times 10^3 = \frac{\pi^2}{16} \times \mu (d_b)^2\, \sigma_t \times n \times d = \frac{\pi^2}{16} \times 0.3\, (d_b)^2\; 70 \times 6 \times 75 = 5830 (d_b)^2$$

$$\therefore \quad (d_b)^2 = 2865 \times 10^3 / 5830 = 492 \quad \text{or} \quad d_b = 22.2 \text{ mm}$$

From Table 11.1, we find that the standard core diameter of the bolt for coarse series is 23.32 mm and the nominal diameter of the bolt is 27 mm (M 27). **Ans.**

13.16 Flange Coupling

Key

A key is a piece of mild steel inserted between shaft and hub or boss of the pulley to connect these together in order to prevent relative motion between them.

→ It is always inserted parallel to the axis of the shaft

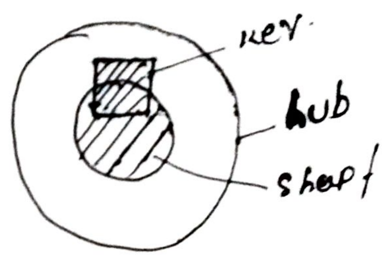
→ Keys are used as temporary fastening and are subjected to considerable crushing and shearing stresses.

Types of keys.

1. Sunk keys
2. Saddle keys
3. Tangent keys
4. Round keys
5. Splines

Sunk keys.

→ The sunk keys are provided half in the keyway of the shaft and half in the keyway of the hub or boss of the pulley



These are following types

→ Rectangular sunk key

→ Square sunk key

→ Parallel sunk key

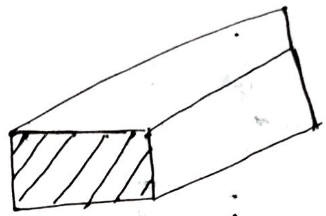
→ Gib - head key.
→ feather key
→ woodruff key

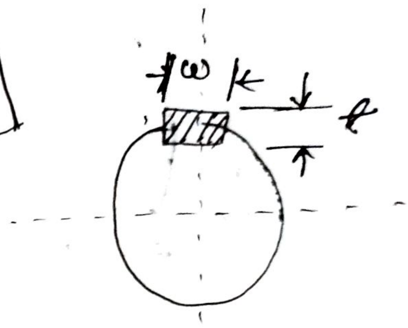
Rectangular key

width key (w) = $\frac{d}{4}$

thickness of key

$t = \frac{2w}{3} = \frac{d}{6}$.



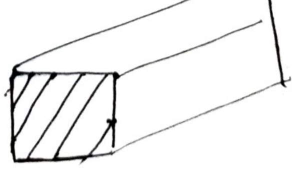


d = Diameter of shaft

Square key

→ The only difference between a rectangular key and square key is that its width and thickness are equal

$$\boxed{w = t = \frac{d}{4}}$$



parallel Sunk key

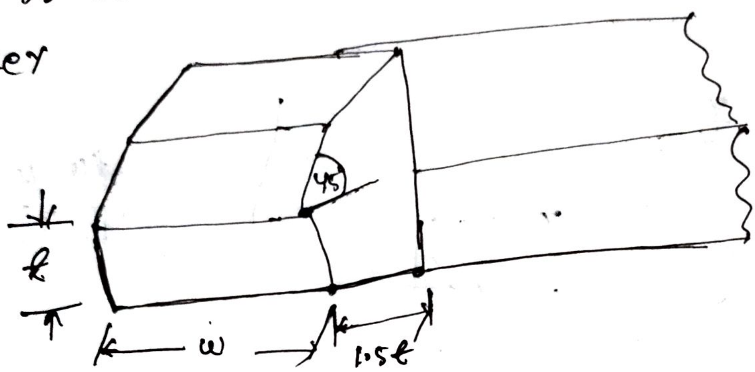
→ The parallel sunk keys may be of rectangular or square section uniform in width and thickness throughout

→ parallel key is a taperless

Gib head key

→ It is a rectangular sunk key with a head at one end known as gib head

→ It is usually provided to facilitate the removal of key



→ width $(w) = \frac{d}{4}$

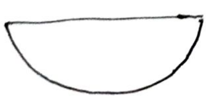
→ Thickness $(t) = \frac{2w}{3} = \frac{d}{6}$

Feather key

→ It allows axial relative motion between hub and shaft

→ It is a special type of parallel key

Woodruff key



→ It is an easily adjustable key.

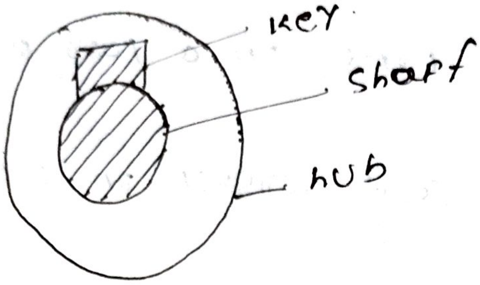
→ It is almost semicircular disk

→ It can be used on tapered shaft

→ Its extra depth in the shaft provide more power transmission.

Saddle key

-> in Saddle key keyway is only in the hub



It is two types

(i) Flat saddle key (ii) Hollow Saddle key.

Flat Saddle key   Hollow Saddle key

Flat Saddle key

-> It is a taper key which fits in a keyway in the hub and is flat on the shaft

-> It is likely to slip round the shaft under load Therefore it is used for comparatively light loads

Hollow Saddle key

-> It is a taper key which fits in a keyway in the hub and the bottom of the key is shaped to fit the curved surface of the shaft.

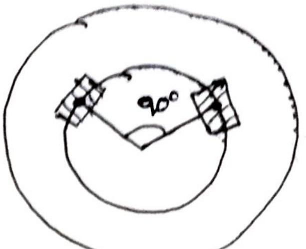
-> Since hollow saddle keys hold on by friction, therefore these are suitable for light loads.

Tangent keys

-> The tangent keys are fitted in a pair at right angles

-> Each key is to withstand torsion in one direction only

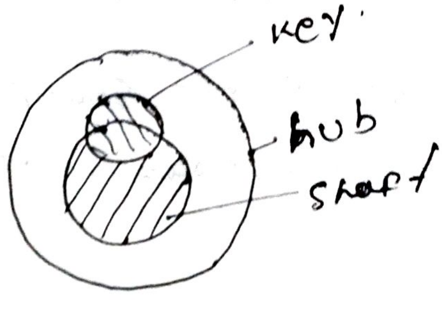
-> These are used in large heavy duty shafts.



Round keys

-> In this key, keyway is created in the shaft and in the hub.

-> It is used for low power transmission.

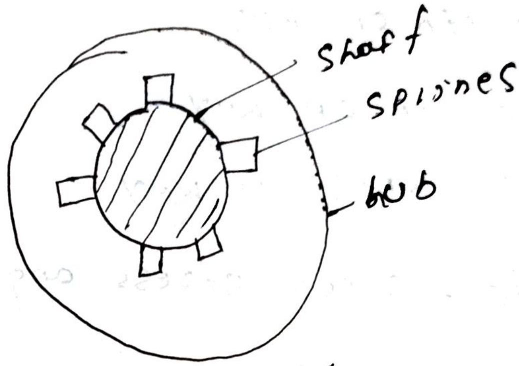


Splines.

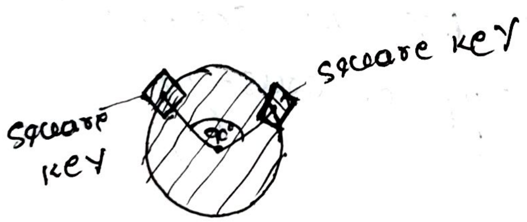
-> Keys are made integral with the shaft which fits in the keyway broached in the hub. Such shaft is known as spindle shaft

→ These shafts usually have four, six, ten or sixteen splines. The splined shafts are relatively stronger than shafts having a single keyway.

→ more power transmission.

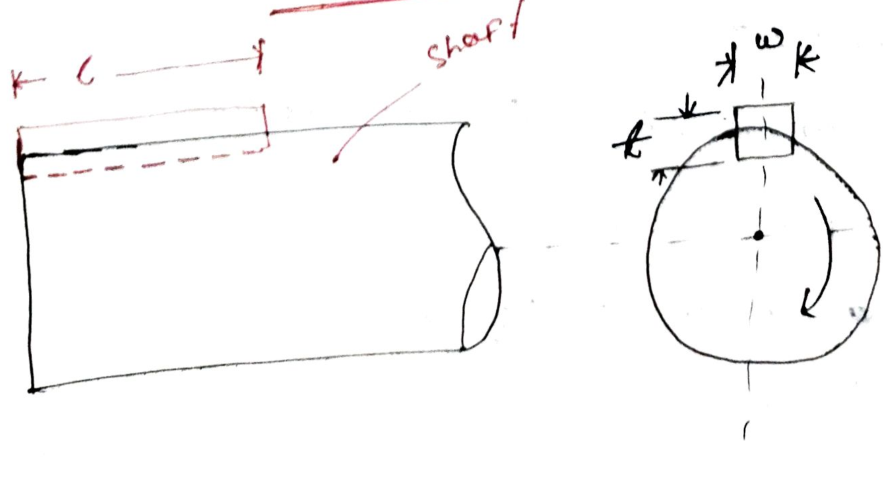


Kennedy key



→ It is used for high power transmission.

Strength of a sunk key



T : Torque transmitted by the shaft

F : Tangential force acting at the circumference of the shaft

d = diameter of shaft

l = length of key

w = width of key

t = Thickness of key.

τ & σ_c = Shear stress and crushing stress for the material of key.

-> A little consideration will show that due to the power transmitted by the shaft, the key may fail due to shearing or crushing.

Considering shearing of the key,

the tangential shearing force acting at the circumference of the shaft

$$F = A \times \tau$$

$$= l \times w \times \tau$$

Torque transmitted by the shaft

$$T = F \times r = F \times \frac{d}{2}$$

$$T = l \times w \times \tau \times \frac{d}{2} \quad \text{——— (1)}$$

Considering crushing of the key,

The tangential crushing force acting at the circumference of the shaft

$$F = A \times \sigma_c = l \times \frac{t}{2} \times \sigma_c$$

Torque transmitted by the shaft

$$T = F \times \frac{d}{2}.$$

$$\boxed{T = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2}.} \quad \text{——— (ii)}$$

equating equation (i) and (ii)

$$l \times w \times \tau \times \frac{d}{2} = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2}$$

$$\boxed{\frac{w}{t} = \frac{\sigma_c}{2\tau}} \quad \text{——— (iii)}$$

$w = t$, a square key is equally strong in shearing and crushing.

we know

Shearing strength of key

$$T = l \times w \times \tau \times \frac{d}{2}. \quad \text{——— (iv)}$$

and torsional shear strength of the shaft

$$T = \frac{\pi}{16} \times \tau_1 \times d^3 \quad \text{——— (v)}$$

(τ_1 = shear stress for shaft material)

From equation (iv) and (v) we get

$$l \times w \times \tau \times \frac{d}{2} = \frac{\pi}{16} \times \tau_1 \times d^3$$

$$l = \frac{\pi}{8} \times \frac{\tau_1 d^2}{w \times \tau} =$$

$$= \frac{\pi d}{8} \times \frac{\tau_1}{\tau} = 1.571\, d \times \frac{\tau_1}{\tau} \quad \left(\text{Taking } w = \frac{d}{4}\right)$$

when the key material is same as that of shaft

then $\tau = \tau_1$

$$\boxed{l = 1.571\, d}$$

Effect of keyways

-> A little consideration will show that the keyway cut into the shaft reduces the load carrying capacity of the shaft

-> This is due to the stress concentration near the corners of the keyway and reduction in the crossectional area of the shaft.

-> In other words, the torsional strength of the shaft is reduced

The following relation for the weakening effect of the keyway is based on the experimental results by H.F moore

$$e = 1 - 0.2\left(\frac{w}{d}\right) - 1.1\left(\frac{h}{d}\right)$$

e = Shaft strength factor. It is the ratio of the strength of the shaft with keyway to the strength of the same shaft without keyway.

w = width of keyway

d = Diameter of shaft

h = Depth of keyway = $\frac{\text{Thickness of key } (t)}{2}$.

Reduction factor for angular twist (K_θ)

$$\boxed{K_\theta = 1 + 0.4\left(\frac{w}{d}\right) + 0.7\left(\frac{h}{d}\right)}$$

When the key material is same as that of the shaft, then ...

$$l = 1.571\, d \qquad \text{... [From equation } (vi)]$$

Example 13.1. *Design the rectangular key for a shaft of 50 mm diameter. The shearing and crushing stresses for the key material are 42 MPa and 70 MPa.*

Solution. Given : $d = 50$ mm ; $\tau = 42$ MPa $= 42$ N/mm^2 ; $\sigma_c = 70$ MPa $= 70$ N/mm^2

The rectangular key is designed as discussed below:

From Table 13.1, we find that for a shaft of 50 mm diameter,

Width of key, $w = 16$ mm **Ans.**

and thickness of key, $t = 10$ mm **Ans.**

The length of key is obtained by considering the key in shearing and crushing.

Let l = Length of key.

Considering shearing of the key. We know that shearing strength (or torque transmitted) of the key,

$$T = l \times w \times \tau \times \frac{d}{2} = l \times 16 \times 42 \times \frac{50}{2} = 16\,800\, l \text{ N-mm} \qquad ...(i)$$

and torsional shearing strength (or torque transmitted) of the shaft,

$$T = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42\,(50)^3 = 1.03 \times 10^6 \text{ N-mm} \qquad ...(ii)$$

From equations (*i*) and (*ii*), we have

$$l = 1.03 \times 10^6 / 16\,800 = 61.31 \text{ mm}$$

Now considering crushing of the key. We know that shearing strength (or torque transmitted) of the key,

$$T = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} = l \times \frac{10}{2} \times 70 \times \frac{50}{2} = 8750\, l \text{ N-mm} \qquad ...(iii)$$

From equations (*ii*) and (*iii*), we have

$$l = 1.03 \times 10^6 / 8750 = 117.7 \text{ mm}$$

Taking larger of the two values, we have length of key,

$$l = 117.7 \text{ say } 120 \text{ mm } \textbf{Ans.}$$

where

Example 13.3. *A 15 kW, 960 r.p.m. motor has a mild steel shaft of 40 mm diameter and the extension being 75 mm. The permissible shear and crushing stresses for the mild steel key are 56 MPa and 112 MPa. Design the keyway in the motor shaft extension. Check the shear strength of the key against the normal strength of the shaft.*

Solution. Given : $P = 15$ kW $= 15 \times 10^3$ W ; $N = 960$ r.p.m. ; $d = 40$ mm ; $l = 75$ mm ; $\tau = 56$ MPa $= 56$ N/mm^2 ; $\sigma_c = 112$ MPa $= 112$ N/mm^2

We know that the torque transmitted by the motor,

$$T = \frac{P \times 60}{2\,\pi N} = \frac{15 \times 10^3 \times 60}{2\,\pi \times 960} = 149 \text{ N-m} = 149 \times 10^3 \text{ N-mm}$$

Let w = Width of keyway or key.

Considering the key in shearing. We know that the torque transmitted (T),

$$149 \times 10^3 = l \times w \times \tau \times \frac{d}{2} = 75 \times w \times 56 \times \frac{40}{2} = 84 \times 10^3\, w$$

$$\therefore \quad w = 149 \times 10^3 / 84 \times 10^3 = 1.8 \text{ mm}$$

This width of keyway is too small. The width of keyway should be at least $d/4$.

$$\therefore \quad w = \frac{d}{4} = \frac{40}{4} = 10 \text{ mm } \textbf{Ans.}$$

Since $\sigma_c = 2\tau$, therefore a square key of $w = 10$ mm and $t = 10$ mm is adopted.

According to H.F. Moore, the shaft strength factor,

$$e = 1 - 0.2\left(\frac{w}{d}\right) - 1.1\left(\frac{h}{d}\right) = 1 - 0.2\left(\frac{w}{d}\right) - 1.1\left(\frac{t}{2d}\right) \quad \ldots (\because h = t/2)$$

$$= 1 - 0.2\left(\frac{10}{20}\right) - \left(\frac{10}{2 \times 40}\right) = 0.8125$$

$\therefore$ Strength of the shaft with keyway,

$$= \frac{\pi}{16} \times \tau \times d^3 \times e = \frac{\pi}{16} \times 56\,(40)^3\;0.8125 = 571\,844 \text{ N}$$

and shear strength of the key

$$= l \times w \times \tau \times \frac{d}{2} = 75 \times 10 \times 56 \times \frac{40}{2} = 840\,000 \text{ N}$$

$$\therefore \quad \frac{\text{Shear strength of the key}}{\text{Normal strength of the shaft}} = \frac{840\,000}{571\,844} = 1.47 \textbf{ Ans.}$$

Shaft.

→ A Shaft is a rotating machine element which is used to transmit power from one place to another.

Axle.

→ an axle is similar in shape to the shaft. it is a stationary machine element and is used for the transmission of bending moment only.
→ it acts as a support for some rotating body.

Spindle.

→ it is a short shaft that imparts motion either to a cutting tool or to a workpiece.

material used for shaft

The material used for shaft should have the following properties.

1. it should have high strength.
2. it should have good machinability.
3. it should have low notch sensitivity factor.
4. it should have good heat treatment properties.
5. it should have high wear resistance properties.

Types of Shaft

1. Transmission shaft.
2. machine shaft.

Design of Shaft

The shaft may be designed on the basis of

① strength ② rigidity and stiffness.

→ In designing shaft on the basis of strength, the following cases may be considered

(a) Shafts subjected to twisting moment or torque only.

(b) Shafts subjected to bending moment only

(c) Shaft subjected to combined twisting and bending moment.

(d) Shaft subjected to axial loads in addition to combined torsional and bending load.

Shafts subjected to twisting moment only.

We know torsion equation

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{L}$$

T = Twisting moment

J = polar moment of inertia.

τ = Shear Stress.

r = radius of the shaft = $\frac{d}{2}$.

G = modulus of rigidity

θ = angle of twist

L = distance.

Considering

$$\frac{T}{J} = \frac{\tau}{r}$$

polar moment of inertia of shaft

$$J = \frac{\pi}{32} \times d^4$$

$$\frac{T}{\frac{\pi}{32} \times d^4} = \frac{\tau}{\frac{d}{2}}$$

$$\boxed{T = \frac{\pi}{16} \times \tau \times d^3}$$

For hollow shaft

$$J = \frac{\pi}{32} \times \left[d_o^4 - d_i^4\right]$$

d_o = outer diameter

d_i = inner diameter

$$\boxed{T = \frac{\pi}{16} \times \tau \times \left[\frac{d_o^4 - d_i^4}{d_o}\right]} \qquad \because K = \frac{d_i}{d_o}$$

$$\boxed{T = \frac{\pi}{16} \times \tau \times d_o^3 \times (1 - K^4)}$$

Power transmitted by shaft in watt

$$P = \frac{2\pi N T}{60} \quad \text{or} \quad T = \frac{P \times 60}{2\pi N}$$

T = Torsion moment in N-m

N = speed of the shaft in RPM

Shafts subjected bending moment only

we know bending equation

$$\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R}$$

M = bending moment

I = moment of inertia

σ_b = bending stress

y = distance from neutral axis to the outer most fibre

R = radius

considering

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

moment of inertia of solid shaft

$$I = \frac{\pi}{64} \times d^4$$

$$y = \frac{d}{2}$$

$$\frac{M}{\frac{\pi}{64} \times d^4} = \frac{\sigma_b}{\frac{d}{2}}$$

$$\boxed{M = \frac{\pi}{32} \times \sigma_b \times d^3}$$

for hollow shaft

$$I = \frac{\pi}{64} \times d_o^4 - d_i^4$$

$$T = \frac{\pi}{32} \times \sigma_b \times \frac{d_o^4 - d_i^4}{d_o}$$

$$\boxed{T = \frac{\pi}{32} \times \sigma_b \times d_o^3 \times (1 - k^4)}$$

shafts subjected to combined twisting moment and bending moment

① maximum shear stress theory or guest's theory.

it is used for ductile material

② maximum normal stress theory or Rankine's theory.

it is used for brittle material.

let τ = shear stress

σ_b = bending stress

According maximum shear stress theory.

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

substituating the value of τ and σ_b.

$$\tau_{max} = \frac{1}{2}\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4\left(\frac{16T}{\pi d^3}\right)^2}$$

$$\tau_{max} = \frac{16}{\pi d^3}\sqrt{M^2 + T^2}$$

$$= \frac{\pi}{16} \times \tau_{max} \times d^3 = \sqrt{M^2 + T^2}$$

where $\sqrt{M^2+T^2}$ = equivalent twisting moment

$$T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16} \times \tau_{max} \times d^3.$$

According to maximum normal stress

$$(\sigma_b)_{max} = \frac{1}{2}\sigma_b + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

$$= \frac{1}{2} \times \frac{32m}{\pi d^3} + \frac{1}{2}\sqrt{\left(\frac{32m}{\pi d^3}\right)^2 + 4 \times \left(\frac{16T}{\pi d^3}\right)^2}$$

$$= \frac{32}{\pi d^3}\left[\frac{1}{2}\left(m + \sqrt{m^2 + T^2}\right)\right]$$

$$= \frac{\pi}{32} \times (\sigma_b)_{max} \times d^3 = \frac{1}{2}\left(m + \sqrt{m^2 + T^2}\right)$$

$\frac{1}{2}\left(m + \sqrt{m^2 + T^2}\right)$ = equivalent bending moment

$$m_e = \frac{1}{2}\left(m + \sqrt{m^2 + T^2}\right) = \frac{\pi}{32} \times (\sigma_b)_{max} \times d^3$$

in case of hollow shaft

$$T_e = \sqrt{m^2 + T^2} = \frac{\pi}{16} \times \tau \times (d_o)^3 \times (1 - k^4)$$

$$m_e = \frac{1}{2}\left(m + \sqrt{m^2 + T^2}\right) = \frac{\pi}{32} \times \sigma_b \times (d_o)^3 \times (1 - k^4)$$

R = Radius of the pulley.

Example 14.1. *A line shaft rotating at 200 r.p.m. is to transmit 20 kW. The shaft may be assumed to be made of mild steel with an allowable shear stress of 42 MPa. Determine the diameter of the shaft, neglecting the bending moment on the shaft.*

Solution. Given : $N = 200$ r.p.m. ; $P = 20$ kW $= 20 \times 10^3$ W; $\tau = 42$ MPa $= 42$ N/mm^2

Let d = Diameter of the shaft.

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 200} = 955 \text{ N-m} = 955 \times 10^3 \text{ N-mm}$$

We also know that torque transmitted by the shaft (T),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\ d^3$$

90/100

$\therefore$ $d^3 = 955 \times 10^3 / 8.25 = 115\ 733$ or $d = 48.7$ say 50 mm **Ans.**

Example 14.2. *A solid shaft is transmitting 1 MW at 240 r.p.m. Determine the diameter of the shaft if the maximum torque transmitted exceeds the mean torque by 20%. Take the maximum allowable shear stress as 60 MPa.*

Solution. Given : $P = 1$ MW $= 1 \times 10^6$ W ; $N = 240$ r.p.m. ; $T_{max} = 1.2\ T_{mean}$; $\tau = 60$ MPa $= 60$ N/mm^2

Let d = Diameter of the shaft.

We know that mean torque transmitted by the shaft,

$$T_{mean} = \frac{P \times 60}{2\pi N} = \frac{1 \times 10^6 \times 60}{2\pi \times 240} = 39\ 784 \text{ N-m} = 39\ 784 \times 10^3 \text{ N-mm}$$

∴ Maximum torque transmitted,

$$T_{max} = 1.2\,T_{mean} = 1.2 \times 39\,784 \times 10^3 = 47\,741 \times 10^3 \text{ N-mm}$$

We know that maximum torque transmitted (T_{max}),

$$47\,741 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 60 \times d^3 = 11.78\,d^3$$

∴ $$d^3 = 47\,741 \times 10^3 / 11.78 = 4053 \times 10^3$$

or $$d = 159.4 \text{ say } 160 \text{ mm} \quad \textbf{Ans.}$$

Example 14.3. *Find the diameter of a solid steel shaft to transmit 20 kW at 200 r.p.m. The ultimate shear stress for the steel may be taken as 360 MPa and a factor of safety as 8.*

If a hollow shaft is to be used in place of the solid shaft, find the inside and outside diameter when the ratio of inside to outside diameters is 0.5.

Solution. Given : $P = 20$ kW $= 20 \times 10^3$ W ; $N = 200$ r.p.m. ; $\tau_u = 360$ MPa $= 360$ N/mm^2 ; F.S. $= 8$; $k = d_i / d_o = 0.5$

We know that the allowable shear stress,

$$\tau = \frac{\tau_u}{F.S.} = \frac{360}{8} = 45 \text{ N/mm}^2$$

Diameter of the solid shaft

Let d = Diameter of the solid shaft.

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 200} = 955 \text{ N-m} = 955 \times 10^3 \text{ N-mm}$$

We also know that torque transmitted by the solid shaft (T),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 45 \times d^3 = 8.84\,d^3$$

∴ $$d^3 = 955 \times 10^3 / 8.84 = 108\,032 \quad \text{or} \quad d = 47.6 \text{ say } 50 \text{ mm} \quad \textbf{Ans.}$$

Diameter of hollow shaft

Let d_i = Inside diameter, and

d_o = Outside diameter.

We know that the torque transmitted by the hollow shaft (T),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau\,(d_o)^3\,(1 - k^4)$$

$$= \frac{\pi}{16} \times 45\,(d_o)^3\,[1 - (0.5)^4] = 8.3\,(d_o)^3$$

∴ $$(d_o)^3 = 955 \times 10^3 / 8.3 = 115\,060 \quad \text{or} \quad d_o = 48.6 \text{ say } 50 \text{ mm} \quad \textbf{Ans.}$$

and $$d_i = 0.5\,d_o = 0.5 \times 50 = 25 \text{ mm} \quad \textbf{Ans.}$$

14.10 Shafts Subjected to Bending Moment Only two values is adopted.

Example 14.5. *A solid circular shaft is subjected to a bending moment of 3000 N-m and a torque of 10 000 N-m. The shaft is made of 45 C 8 steel having ultimate tensile stress of 700 MPa and a ultimate shear stress of 500 MPa. Assuming a factor of safety as 6, determine the diameter of the shaft.*

Solution. Given : $M = 3000$ N-m $= 3 \times 10^6$ N-mm ; $T = 10\ 000$ N-m $= 10 \times 10^6$ N-mm
$\sigma_{tu} = 700$ MPa $= 700$ N/mm^2 ; $\tau_u = 500$ MPa $= 500$ N/mm^2

We know that the allowable tensile stress,

$$\sigma_t \text{ or } \sigma_b = \frac{\sigma_{tu}}{F.S.} = \frac{700}{6} = 116.7 \text{ N/mm}^2$$

and allowable shear stress,

$$\tau = \frac{\tau_u}{F.S.} = \frac{500}{6} = 83.3 \text{ N/mm}^2$$

Let d = Diameter of the shaft in mm.

According to maximum shear stress theory, equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(3 \times 10^6)^2 + (10 \times 10^6)^2} = 10.44 \times 10^6 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$10.44 \times 10^6 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 83.3 \times d^3 = 16.36\ d^3$$

$$\therefore \quad d^3 = 10.44 \times 10^6 / 16.36 = 0.636 \times 10^6 \text{ or } d = 86 \text{ mm}$$

According to maximum normal stress theory, equivalent bending moment,

$$M_e = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) = \frac{1}{2}(M + T_e)$$

$$= \frac{1}{2}(3 \times 10^6 + 10.44 \times 10^6) = 6.72 \times 10^6 \text{ N-mm}$$

We also know that the equivalent bending moment (M_e),

$$6.72 \times 10^6 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 116.7 \times d^3 = 11.46\, d^3$$

$$\therefore \quad d^3 = 6.72 \times 10^6 / 11.46 = 0.586 \times 10^6 \text{ or } d = 83.7 \text{ mm}$$

Taking the larger of the two values, we have

$$d = 86 \text{ say } 90 \text{ mm}$$ **Ans.**

Spring

→ A spring is defined as an elastic body, whose function is to distort when loaded and to recover its original shape when the load is removed.

Various application of spring

→ To cushion, absorb or control energy due to either shock or vibration.

→ To apply forces, as in brakes, clutches and spring loaded valves.

→ To control motion by maintaining contact between two elements as in cams and followers.

→ To measure forces, as in spring balances and engine indicators.

→ To store energy, as in watches, toys etc

Types of Spring

(i) Helical spring
(ii) Conical and volute spring
(iii) Torsion spring
(iv) Leaf spring
(v) Disc or belleville spring
(vi) Special purpose spring.

Material for helical spring

→ The material of the spring should have high fatigue strength, high ductility, high resilience, and it should be creep resistant.

Term used in compression spring

① Solid length

→ It is the product of total number of coil and diameter of the wire.

Mathematically

$$\boxed{L_s = n' d}$$

L_s = Solid length

n' = Total number of coil

d = Diameter of the wire.

② Free length (L_F)

→ It is equal to the solid length plus the maximum deflection or compression of the spring and the clearance between the adjacent coil

Mathematically

L_F = Solid length + maximum compression + clearance between the adjacent coils.

$$\boxed{L_f = n'd + \delta_{max} + 0.15\,\delta_{max}}$$

The following relation may also used to find the free length of spring

$$\boxed{L_f = n'd + \delta_{max} + (n'-1) \times 1\ mm}$$

In this expression, The clearance between the two adjacent coils is taken as 1 mm.

③ Spring index (C)

→ The spring index is defined as the ratio of the mean diameter of the coil to the diameter of the wire

Mathematically

$$\boxed{C = \frac{D}{d}}$$

D = mean diameter of the coil

d = Diameter of the wire.

④ Spring Rate. (K)

→ It is the ratio of load required per unit deflection of the spring.

$$\boxed{K = \frac{W}{\delta}}$$

W = load

δ = Deflection of the spring

(5) pitch.

→ The pitch of the coil is defined as the axial distance between adjacent coil in uncompressed state.

mathematically

$$P = \frac{\text{free length}}{n' - 1}$$

→ The pitch of the coil may also obtained by using the following relation

$$\boxed{P = \frac{L_f - L_s}{n'} + d}$$

L_f = free length of the spring

L_s = solid length of the spring

n' = Total number of coil

d = Diameter of wire.

Stresses in helical spring of circular wire

D = mean diameter of coil

d = diameter of wire

n = No. of turn of coil

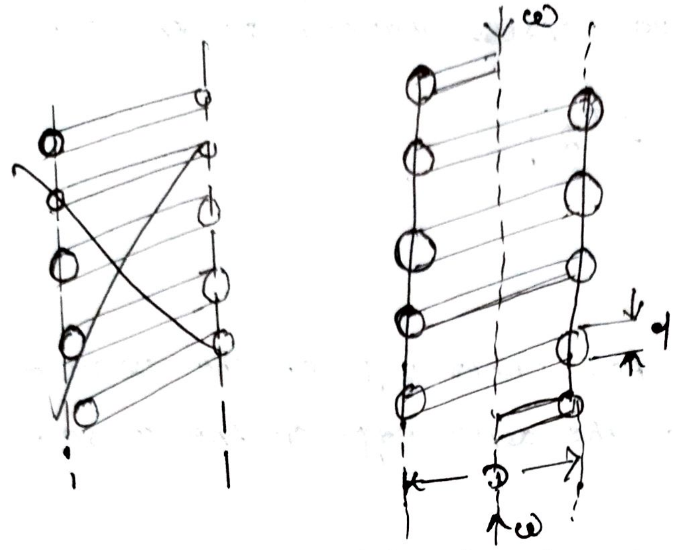
G = modulus of rigidity

W = Axial load

C = spring index

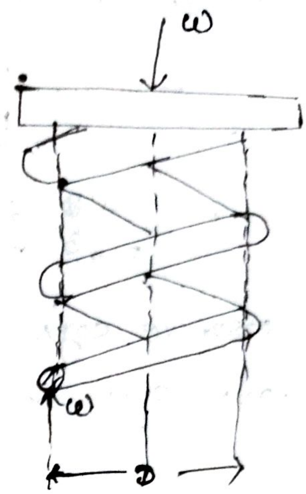
P = pitch of the coil

δ = Deflection of the spring



(a)

(Axially loaded helical spring)



(b)

(wire subjected to torsional shear and a direct shear)

we know twisting moment

$$T = w \times \frac{D}{2} = \frac{\pi}{16} \times \tau_1 \times d^3$$

$$\tau_1 = \frac{8 w D}{\pi d^3}$$

τ_1 = torsional shear stress

The following stresses also act on the wire

① Direct shear stress due to the load (w)

② Stress due to the curvature of wire.

we know that direct shear stress due to the load (w)

$$\tau_2 = \frac{\text{load}}{\text{cross-sectional area of the wire}}$$

$$= \frac{w}{\frac{\pi}{4} \times d^2} = \frac{4w}{\pi d^2}$$

So that the resultant shear stress induced in the wire.

$$\boxed{\tau = \tau_1 \pm \tau_2}$$

$$= \frac{8WD}{\pi d^3} \pm \frac{4W}{\pi d^2}$$

The positive sign is used for inner edge of the wire and negative sign is used for the outer edge of the wire.

maximum shear stress induced in the wire

Torsional shear stress + Direct shear stress

$$\frac{8WD}{\pi d^3} + \frac{4W}{\pi d^2} = \frac{8WD}{\pi d^3}\left(1 + \frac{d}{2D}\right)$$

$$= \frac{8WD}{\pi d^3}\left(1 + \frac{1}{2C}\right)$$

$$= K_S \times \frac{8WD}{\pi d^3} \qquad \because \frac{D}{d} = C$$

where K_S = shear stress factor

$$K_S = \left(1 + \frac{1}{2C}\right)$$

maximum shear stress induced in the wire

$$\tau = K \times \frac{8WD}{\pi d^3} = K \times \frac{8WC}{\pi d^2}$$

where $K = \frac{4C-1}{4C-4} + \frac{0.615}{C}$ → called wahl's factor

wahl's stress factor (K) may be composed of two sub-factors K_S and K_C

$$\boxed{K = K_S \times K_C}$$

K_S = stress factor due to shear

K_C = stress concentration factor.

Deflection of Helical Spring of circular wire.

Total active length of the wire.

L = length of one coil x No. of active coil

$= \pi D \times n$

θ = Angular Deflection of the wire when acted upon by the torque T

let

Axial deflection of spring

$$\delta = \theta \times \frac{D}{2}.$$

we know Torque equation

$$\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L}$$

$$\frac{T}{J} = \frac{G\theta}{L}$$

$$\theta = \frac{TL}{G \cdot J}$$

J = polar moment of inertia

$$J = \frac{\pi}{32} \times d^4$$

d = diameter of spring wire

G = modulus of Rigidity.

now substituting the value of 'L' and J in above equation

$$\theta = \frac{TL}{J \cdot G} = \frac{\left(W \times \frac{D}{2}\right) \pi D \cdot n}{\frac{\pi}{32} \times d^4 \cdot G} = \frac{16 W \cdot D^2 \cdot n}{G d^4}$$

$$\delta = \theta \times \frac{D}{2}$$

$$= \frac{16 W \cdot D^2 \cdot n}{G d^4} \times \frac{D}{2} = \frac{8 W \cdot D^3 \cdot n}{G d^4}$$

$$= \frac{8 \cdot W \cdot C^3 \cdot n}{G \cdot d} \quad \left(\because C = \frac{D}{d}\right)$$

Spring rate or stiffness of the spring

$$\frac{W}{\delta} = \frac{G d^4}{8 D^3 n} = \frac{G d}{8 C^3 n} = \text{constant}$$

Eccentric loading of spring

→ Sometimes, The load on the spring does not coincide with the axis of the spring i.e the spring is subjected to an eccentric load.

→ in such case, not only the safe load for the spring reduces, The stiffness of the spring is also affected then the safe load on the spring may be obtained by multiplying the axial load by the factor

$\frac{D}{2e+D}$, where D is the mean Diameter of spring.

Buckling of compression spring

The critical axial load (W_{cr}) that causes buckling may be calculated by using following Relation

$$\boxed{W_{cr} = K \times K_B \times K_f}$$

K = Spring Rate or Stiffness of the spring $= \frac{W}{\delta}$

L_f = free length of the spring

K_B = Buckling factor depending upon the ratio $\frac{L_f}{D}$

Surge in spring

→ When one end of a helical spring is resting on a rigid support and other end is loaded suddenly, then all the coils of the spring will not suddenly deflect equally, because some time is required for the propagation of stress along the spring wire. A little consideration will show that in the beginning, the end coils of the spring in contact with applied load takes up whole of the deflection and then it transmits a large part of its deflection to the adjacent coils.

This phenomenon is called surge in spring.

→ It has been found that the natural frequency of spring should be atleast twenty times the frequency of application of a periodic load in order to avoid resonance with all harmonic frequencies upto twentieth order.

The natural frequency for spring clamped between two plates is given by

$$f_n = \frac{d}{2\pi D^2 n}\sqrt{\frac{G g}{\rho}} \text{ cycle/s}$$

d = Diameter of wire.

D = mean diameter of spring

n = number of active turn

G = modulus of rigidity

g = Acceleration due to gravity.

ρ = Density of the material of the spring different natural frequencies.

Example 23.1. *A compression coil spring made of an alloy steel is having the following specifications :*

Mean diameter of coil = 50 mm ; Wire diameter = 5 mm ; Number of active coils = 20.

If this spring is subjected to an axial load of 500 N ; calculate the maximum shear stress (neglect the curvature effect) to which the spring material is subjected.

Solution. Given : $D = 50$ mm ; $d = 5$ mm ; *$n = 20$; $W = 500$ N

We know that the spring index,

$$C = \frac{D}{d} = \frac{50}{5} = 10$$

∴ Shear stress factor,

$$K_S = 1 + \frac{1}{2C} = 1 + \frac{1}{2 \times 10} = 1.05$$

and maximum shear stress (neglecting the effect of wire curvature),

$$\tau = K_S \times \frac{8W.D}{\pi\, d^3} = 1.05 \times \frac{8 \times 500 \times 50}{\pi \times 5^3} = 534.7 \text{ N/mm}^2$$

$$= 534.7 \text{ MPa} \quad \textbf{Ans.}$$

Example 23.2. *A helical spring is made from a wire of 6 mm diameter and has outside diameter of 75 mm. If the permissible shear stress is 350 MPa and modulus of rigidity 84 kN/mm², find the axial load which the spring can carry and the deflection per active turn.*

Solution. Given : $d = 6$ mm ; $D_o = 75$ mm ; $\tau = 350$ MPa $= 350$ N/mm² ; $G = 84$ kN/mm² $= 84 \times 10^3$ N/mm²

We know that mean diameter of the spring,

$$D = D_o - d = 75 - 6 = 69 \text{ mm}$$

$\therefore$ Spring index,

$$C = \frac{D}{d} = \frac{69}{6} = 11.5$$

Let

$$W = \text{Axial load, and}$$

$$\delta / n = \text{Deflection per active turn.}$$

1. *Neglecting the effect of curvature*

We know that the shear stress factor,

$$K_S = 1 + \frac{1}{2C} = 1 + \frac{1}{2 \times 11.5} = 1.043$$

and maximum shear stress induced in the wire (τ),

$$350 = K_S \times \frac{8\,W.D}{\pi\,d^3} = 1.043 \times \frac{8\,W \times 69}{\pi \times 6^3} = 0.848\,W$$

$$\therefore \quad W = 350 / 0.848 = 412.7 \text{ N } \textbf{Ans.}$$

We know that deflection of the spring,

$$\delta = \frac{8\,W.D^3.n}{G.d^4}$$

$\therefore$ Deflection per active turn,

$$\frac{\delta}{n} = \frac{8\,W.D^3}{G.d^4} = \frac{8 \times 412.7\,(69)^3}{84 \times 10^3 \times 6^4} = 9.96 \text{ mm } \textbf{Ans.}$$

2. *Considering the effect of curvature*

We know that Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 11.5 - 1}{4 \times 11.5 - 4} + \frac{0.615}{11.5} = 1.123$$

We also know that the maximum shear stress induced in the wire (τ),

$$350 = K \times \frac{8W.C}{\pi\,d^2} = 1.123 \times \frac{8 \times W \times 11.5}{\pi \times 6^2} = 0.913\,W$$

$$\therefore \quad W = 350 / 0.913 = 383.4 \text{ N } \textbf{Ans.}$$

and deflection of the spring,

$$\delta = \frac{8\,W.D^3.n}{G.d^4}$$

$\therefore$ Deflection per active turn,

$$\frac{\delta}{n} = \frac{8\,W.D^3}{G.d^4} = \frac{8 \times 383.4\,(69)^3}{84 \times 10^3 \times 6^4} = 9.26 \text{ mm } \textbf{Ans.}$$