STRUCTURAL MECHANICS

TH-1

3rd SEM

CIVIL ENGG.

Under SCTE&VT,Odisha

PREPARED BY:-

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MECHANICS OF MATERIALS

3RD SEMESTER SUBJECT CODE –CET 301

TOPICS TO BE COVERED

- ***REVIEW OF BASIC CONCEPTS***
- ***SIMPLE STRESSE & STRAINS***
- ***APPLICATION OF STRESS & STRAIN IN ENGINEERING FIELD***
- ***GEOMETRICAL PROPERTIES OF SECTIONS FOR CENTRE OF GRAVITY***
- ***GEOMETRICAL PROPERTIES OF MOMENT OF INERTIA***
- ***SHEAR FORCE & BENDING MOMENT***
- ***STRESSES IN BEAMS DUE TO BENDING***
- ***SHEAR STRESSES IN BEAMS***
- ***STRESSES IN SHAFTS DUE TO TORSION***
- ***COMBINED BENDING & DIRECT STRESSES***
- ***COMPLEX STRESSES & STRAINS***

TOTAL MARKS =100
CLASS TEST =20
TEACHER ASSESSMENT =10
END SEMESTER EXAM=70

- ***A TEXT BOOK OF STRENGTH OF MATERIAL –R.S.KHURMI***
- ***REFERENCE BOOK –STRENGTH OF MATERIAL –R.K.RAJPUT***
- ***REFERENCE BOOK-STRENGTH OF MATERIAL-S.RAMAMURTHAM***

CHAPTER-1 **REVIEW OF BASIC CONCEPTS**

TOPICS TO BE COVERED

- **INTRODUCTION**
- **BASIC PRINCIPLE OF MECHANICS**
- **FORCE**
- **MOMENT**
- **EQUILIBRIUM**
- **BODY CONSTRAINTS**
- **FREE BODY DIAGRAM**

INTRODUCTION

What Is Mechanics ?

Mechanics may be defined as the science , which describes & predicts the conditions of rest or motion of bodies under the action of forces.

What Is Engg Mechanics ?

Engg mechanics is the branch of engineering that applies the principles of mechanics to which must be take in to account the effect of forces.

Definition Of Force.

Force is defined as an external agent which changes or tends to change the state of rest or of uniform motion of a body along a straight line.

Ex:-If a force is applied to a paper, weight resting on a table , the paper weight will move or will have a tendency to move from its state of rest.

What Is Moment ?

It is the turning effect produced by a force, on the body , on which it acts . The moment of a force is equal to the product of the force & the perpendicular distance of the point , about which the moment is required & the line of action of the force.

Mathematically $M = p \times l$

Where p=force acting on the body

L=perpendicular distance between the point , about which the moment is required & the line of action of the force.

What Is Equilibrium ?

When a particle is acted upon by a no. Of forces .If the resultant of a no. of forces , acting on a particle is zero.The particle will be in equilibrium.

Such a set of forces whose resultant is zero are called equilibrium forces.

Conditions For Equilibrium:-

- The body may move in any direction

  $\sum F_h = 0$ & $\sum F_v = 0$

- The body may be completely at rest

  $\sum M = 0$ $\sum F_h = 0$ & $\sum F_v$

Definition Of Body Constraints:-

The conditions of equilibrium of bodies that are not entirely free to move .Restriction to the free motion of a body in any direction is called constraint.

Free Body:-

A free body is a body not connected with other bodies & which from any given position can be displaced in any direction in space.

Free Body Diagram:-

We shall always imagine that we remove the supports & replace them by the reactions which they exert on the body .

*Free body diagram is a sketch of the isolated body , which shows the external forces on the body & the reactions exerted on it by the removed elements.

CHAPTER -2 **SIMPLE STRESSES & STRAINS**

TOPICS TO BE COVERED

- *MECHANICAL PROPERTIES OF MATERIALS*
- SOME TERMS OF DEFINATION
- DEFINATION OF STRESS & STRAIN
- TYPES OF STRESSS & STRAINS
- LONGITUDINAL & LATERAL STRAINS AND POISSON'S RATIO
- VOLUMETRIC STRAIN, HOOKE'S LAW ELASTIC CONSTANTS
- RELATIONSHIP BETWEEN ELASTIC CONSTANTS
- YOUNG'S MODULUS VALUES FOR ENGG MATERIALS

deformation:- a change in the dimension of object or changes of an object shape or force into a
Bending:- shape or curve or angle Ex:- road, pipe etc.
Twist:- Form into a bent, → curling → distorted shape → cause to rotate around a fixed pt. → changes of direction
Stretching:- It has capable of longer → ... wider without tearing or breaking.

Some terms of Definition

Rigidity :-

It is defined as the property of a solid body to resists to deformation , bending , twisting, stretching under a load.

Compressibility :-

It is a measure of the relative volume change of the fluid or solid as a response to a pressure (or mean stress) change.

Hardness:-

It is defined as the ability of a material to resists plastic deformation usually by indentation.

Toughness:-

It is the ability of a material to absorb energy & plastically deform without fracturing.

*It is the amount of energy per volume that a material can absorb before rupturing.

Stiffness:-

It is the rigidity of an object , the extent to which it resists deformation in response to an applied force.

Creep:-

It is defined as the form of plastic deformation that takes place in steel held for long periods at high temperature.

Fatigue:-

It is defined as the effect on metal of repeated cycles of stress. The insidious feature of fatigue failure is that there is no obvious warning , a crack forms without appreciable deformation of structure.

Durability:-

It is long continuous useful life .The ability to withstand wear, pressure or damage etc. Well lasting enduring. → last

Elasticity:-

A material is said to be perfectly elastic if the whole strain produced by a load disappears completely on the removal of the load. OR The ability of a material to deform under load & return to its original shape when the load is removed is called elasticity.

Plasticity:-

It is defined as the property that enables the formation of a permanent deformation in a material. It is the reverse of elasticity. OR The ability of a material to deform under load & retain its new shape when the load is removed is called Plasticity.

Ductility:-

It is the ability of a metal to withstand elongation or bending.

Fatigue:- Failure of material under repeated or reversal stresses is called fatigue.
Strength:- It is defined as the ability of a material to resist loads without failure.
Tensile strength:- It is defined as the ability of material to resist a stretching (tensile) load without fracture.

Malleability:-

It is defined as the property by virtue of which a material may be hammered or rolled in to thin sheets without rupture.

Brittleness:-

When a body breaks easily when subjected to shock it is said to be brittle.

Tenacity:-

It is the strength with which the material opposes rupture.

Definition Of Stress

Whenever some external system of forces acts on a body, it undergoes some deformation . As the body undergoes deformation its molecules set up some resistance to deformation. This deformation per unit area to deformation is known as stress.

Or

The internal resistance which the body offers to meet with the load is called stress.

Mathematically, stress may be defined as the force per unit area is called stress.

It is denoted by letter σ

$\sigma = P/A$

Where P=Load or force acting on the body.

A=c/s area of the body.

Strain:-

The strain is the deformation produced by stress. The deformation per unit length is known as strain.

Mathematically, strain may be defined as the deformation per unit length is called strain.

It is denoted by letter ε

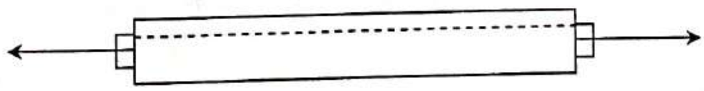
$$\varepsilon = \delta l / l$$

Where δ l= change in length of the bar

L = original length of the bar

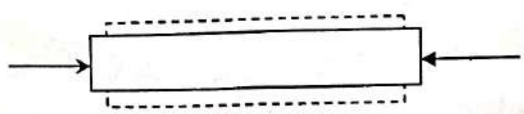
Tensile Stress:-

When a section is subjected to two equal & opposite pulls & the body tends to increase its length, the stress induced is called tensile stress.



Compressive Stress:-

When a section is subjected to two equal & opposite pushes & the body tends to shorten its length, the stress induced is called compressive stress.



Shear Stress:-

When a section is subjected to two equal & opposite forces, acting tangentially across the resisting section, as a result of which the body tends to shear off across the section, the stress induced is called shear stress.

Modulus Of Elasticity Or Young 'S Modulus:-

Whenever a material is loaded within its elastic limit, the stress is directly proportional to strain.

Mathematically, $\sigma \propto \varepsilon$

$$=E\times\varepsilon$$

Or, $E=\sigma/\varepsilon$

Where σ= stress

ε=strain

E= A constant of proportionality known as modulus of elasticity or young's modulus.

Deformation of a body due to force acting on it:-

Consider a body subjected to a tensile stress.

Let P=Load or force acting on the body,

L=Length of the body,

A=Cross-sectional area of the body,

σ=Stress induced in the body,

E=Modulus of elasticity for the bar material of the body,

ε=Strain, &

δl=Deformation of the body.

We know that the stress

$$\sigma = P/A$$

$$\text{Strain } \varepsilon = \sigma/E = P/AE$$

& Deformation, $\delta l = \varepsilon \times l$

$$=\sigma l/E = Pl/AE$$

Example -1

A steel rod 1m long and 20mm×20mm in cross-section is subjected to a tensile force of 40KN. Determine the elongation of the, if modulus of elasticity for the rod material is 200Gpa.

Solution:-

Given data

Length $l=1m=1\times10^3$ mm

Cross-sectional area (A)=20×20 =400mm^2

Tensile force (P)=40KN=40×10^3N

Modulus of elasticity E=200Gpa=$200\times10^3 N/mm^2$

We know that elongation of the rod,

δl= pl/AE

$$= \frac{(40\times10^3)\times(1\times10^3)}{(400\times(20\times10^3)}$$

= 0.5mm

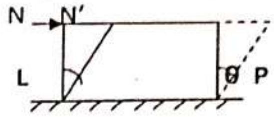
Types Of Strains :-

Tensile Strains:- Apiece of material, with uniform cross-section, subjected to a uniform axial tensile stress, will increase its length from l to $(l + \delta l)$ & the increment of length δl is the actual deformation of the material.

The tensile strains $\varepsilon_t=\delta l/l$

Compressive Strain: - Under compressive forces ' a similar piece of material would be reduced in length from l to $(l-\delta l)$
The strain $\varepsilon_c = \delta l/l$

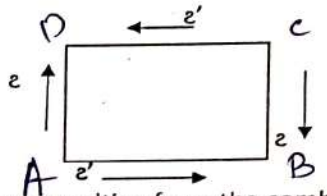
Shear Strain: - In case of a shearing load , a shear strain will sb produced which is measured by an angle through which the body distorts.   F M M'



Shear strain ε_s = NN'/NP = tan θ

Complementary Shear Stress :- Whenever a shear stress ε is applied on parallel surface of the body then to keep in equilibrium a shear stress 'ε'' is induced on remaining surface of the body . These stresses form a couple .The couple from due to shear stress ε produces clockwise moment .For equilibrium this couple is balanced by couple developed by ε' .This resisting shear stress ε is known as complementary shear stress.

Couple produced by ε
(ε BC)×AB
Coupled produced by ε'   ε'(CD)×BC



For equilibrium (ε ×BC)×AB = (ε'×CD)×BC
⇨ ε = ε' {AB = CD }

Diagonal Tensile Stress: - One of the principal stresses resulting from the combination of horizontal & vertical shear stresses in a beam or slab . In brittle materials such as concrete it results in diagonal cracks.

Longitudinal Strain: - It is defined as the ratio between stress to the modulus of elasticity of material. It is denoted by letter 'ε'
ε=stress/E

Linear Strain: - Whenever some external force acts on a body it undergoes some deformation .The deformation of the bar per unit length in the direction of the force i.e. δl/l is known as linear strain.

Lateral Strain :- Whenever some external force acts on a bocy it undergoes some deformation .The deformation of the bar has extended through a length δl , which will decrease of diameter from d to (d+δd).The strain is known as lateral strain.

Poisson's Ratio: - If a body is stressed within its elastic limit , the lateral strain bears a constant ratio to the linear strain.
Mathematically,   Lateral strain/ Linear strain
=constant
This constant is known as poison's ratio & it is denoted by 1/m or μ

Mathematically, Lateral strain = 1/m × ε
⇨ Lateral strain =1/m × linear strain (δl/l)
⇨ Linear strain=1/m × lateral strain (δb/b)
⇨ Linear strain= 1/m × δb/b

Volumetric Strain :- It is defined as the ratio of change in volume to the original volume is known as volumetric strain.
Mathematically, $\varepsilon_v = \delta v/v$
Where δv = Change in volume
V=Original volume

Hooke's Law: - It states that 'When a material is loaded within its elastic limit , the stress is proportional to the strain.

Mathematically,

Stress/Strain = E

Elastic Constants

Young's Modulus Of Elasticity: - Whenever a material is loaded , within its elastic limit, the stress is proportional to strain .

Mathematically,

$\sigma \alpha \varepsilon$

$=E \times \varepsilon$

Where E=A constant of proportionality known as modulus of elasticity or young's modulus.

Shear Modulus Or Modulus Of Rigidity :-

Within its elastic limit , the shear stress is proportional to the shear strain.

Mathematically, $\tau \alpha \phi$

$\Rightarrow \tau = C \times \phi$

Where τ = Shear stress

ϕ = Shear strain

C= A constant known as shear modulus or modulus of rigidity.

Bulk Modulus Or Modulus Of Compressibility :-

When a body is subjected to three mutually perpendicular stresses of equal intensity, the ratio of direct stress to the corresponding volumetric strain is known as bulk modulus. It is denoted by K

Mathematically K = $\frac{\text{Direct stress}}{\text{Volumetric strain}}$

$= \sigma / (\delta V/V)$

Young's Modulus Values For Materials :-

Sl. No.	Material	Modulus Of Elasticity (E) In Gpa I.E. GN/M^2 Or KN/M^2
1.	Steel	200-220
2.	Wrought iron	190-200
3.	Cast iron	100-160
4.	Copper	90-110
5.	Brass	80-90
6.	Aluminium	60-80
7.	Timber	10

Problem -1

A steel bar 2m long, 40 mm wide & 20mm thick is subjected to an axial pull of 160 KN in the direction of its length . Find the changes in length, width, & thickness of the bar. Take E=200 Gpa and poison's ratio =0.3

Solution :-

Given data:-

Length l = 2m = 2×10^3mm

Width b = 40mm

Thickness t= 20mm

Axial pull P=160 KN =160×10^3 N

Modulus of elasticity E = 200Gpa =200×10^3 N/mm^2

Poisson's ratio 1/m =0.3

Change in length

We know that change in length

$\delta l = Pl/AE$

$\dfrac{(160\times10^3)\times(2\times10^3)}{(40\times20)\times(200\times10^3)}$

$= 2$ mm

Change in width:-

We know that linear strain $\varepsilon = \delta l/l$

$= 2/(2\times10^3)$

$= 0.001$

And lateral strain $= 1/m \times \varepsilon$

$= 0.3 \times 0.01$

$= 0.0003$

Hence change in width, $\delta b = b \times$ lateral strain

$= 40 \times 0.0003$

$= 0.012$ mm

Change in thickness, $\delta t = t \times$ lateral strain

$= 20 \times 0.0003$

$= 0.006$ mm

Problem -2

A concrete cylinder of diameter 150 mm & length 300 mm when subjected to an axial compressive load of 240 KN resulted in an increase of diameter by 0.127 mm & a decrease in length of 0.28 mm. Compute the value of poison's ratio μ or $1/m$ & modulus of elasticity E

Solution:-

Given data

Diameter of the cylinder d = 150 mm

Length of the cylinder l = 300 mm

Increase in diameter $\delta d = 0.127$mm

Decrease in length $\delta l = 0.28$ mm

Axial compressive load P = 240 KN

We know that

Linear strain $= \delta l/l$

$= 0.28/300$

$= 0.000933$

& Lateral strain $= \delta d/d$

$= 0.127/150$

$= 0.000846$

Poisson's ratio $\mu = \dfrac{\text{Lateral strain}}{\text{Linear strain}}$

$= \dfrac{0.000846}{0.000933}$

$= 0.907$

Modulus of elasticity $E = \dfrac{\text{Stress}}{\text{Linear strain}}$

$= \dfrac{P/A}{\delta L/l}$

$= \dfrac{\dfrac{(240\times10^3)}{\pi/4\times(150)^2}}{0.000933}$

$= 14.55\times10^6$ KN/m^2

$= 14.55$ GN/m^2

Relation Between Bulk Modulus & Young's Modulus :-

Consider a cube ABCD EFGH

Let the cube be subjected to three mutually perpendicular tensile stresses acting on a cube.

Let σ = Stress on the faces

l= Length of the cube

E = Young's modulus for the material of the cube

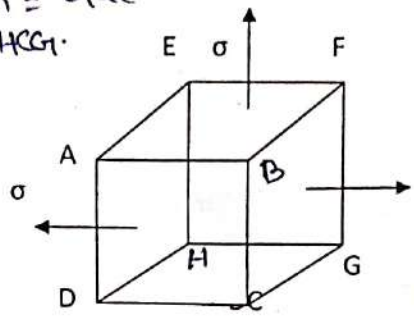
When stress acting on horizontally direction, the tensile strain = σ/E, due to stresses on the faces BF, CG & AE, DH.

When stress acting on vertically direction AB & DC line EF & GH line will be shorten.

This stresses is known as compressive lateral strain.

Poisson's ratio $\mu = \dfrac{\text{Lateral strain}}{\text{Linear strain}}$ → comp. lat. strain = due to stresses on faces AE BF & DHCG.

$= \dfrac{\text{Compressive lateral strain}}{\text{Strain}}$



$\Rightarrow \mu = \dfrac{\text{Compressive lateral strain}}{\sigma/E}$

$\Rightarrow$ Compressive lateral strain = $-(\mu \times \sigma/E)$

Strain of AB = σ/E

$\mu = \dfrac{\text{Compressive lateral strain}}{\sigma/E}$

Compressive lateral strain = $-(\mu \times \sigma/E)$

Strain σ/E, $(-\mu \times \sigma/E)$, $(-\mu \times \sigma/E)$

Total strain AB $= \sigma/E + (-\mu \times \sigma/E) + (-\mu \times \sigma/E)$

$= \sigma/E - \mu \times \sigma/E - \mu \times \sigma/E$

$= \sigma/E\,(1-\mu-\mu)$

$= \sigma/E\,(1-2\mu)$

$= \delta l/l$

$\left[y = x^3 \Rightarrow \dfrac{dy}{dx} = 3x^2\right]$

Total volume $V = l^3$

$\delta v/\delta l = 3l^2$ $(\because v = l^3)$

$\delta v = 3l^2 \times \delta l$

$\delta v/v = 3l^2/l^3 \times \delta l$

$\Rightarrow \delta v/v = 3(\delta l/l)$

$\Rightarrow \delta v/v = 3 \times \sigma/E(1-2\mu)$

Bulk modulus $K = \sigma/(\delta v/v)$

$= \sigma / 3\times\sigma/E\,(1-2\mu)$

$= E/3(1-2\mu)$

$= E/3(1-2/m)$

$= E/3(m-2/m) =$ $K = \dfrac{mE}{3(m-2)}$

Problem -1

If the values of modulus of elasticity and poisson's ratio for an alloy body are 150Gpa and 0.25 respectively, determine the value of bulk modulus for the alloy.

Solution:-

Modulus of elasticity E =150 Gpa = 150×10^3 N/mm^2

And poisson's ratio (1/m)= 0.25

Or m = 4

We know that value of the bulk modulus for the alloy,

$K = mE/3(m-2)$

$$= \frac{4 \times (150\times10^3)}{3(4-2)}$$

$=100 \times10^3$ N/mm^2

=100Gpa

Problem -2

For a given material , young's modulus is 120Gpa and modulus of rigidity is 40Gpa .Find the bulk modulus and lateral contraction of a round bar of 50mm diameter and 2.5 m long ,when stretched 2.5 mm . Take poisson's ratio as 0.25.

Solution:-

Given data:-

Young's modulu's (E) = 120 Gpa $=120 \times10^3$ N/mm^2

Modulus of rigidity (C) =40 Gpa = 40×10^3 N/mm^2

Diameter (D) = 50mm

Length l =2.5m $=2.5\times 10^3$ mm

Change in length δl =2.5 mm

Poisson's ratio m = 0.25 or m = 4

Bulk modulus of the bar $K = \frac{mE}{3(m-2)}$

$$= \frac{4\times(120\times10^3)}{3(4-2)}$$

$= 80 \times 10^3$ N/mm^2

Lateral contraction of the bar:-

We know that

Linear strain $\varepsilon = \delta l/l$

$=2.5/ (2.5\times10^3)$

$= 0.001$

And lateral strain $\delta d/d = 1/m \times \varepsilon$

$= 0.25\times 0.001$

$=0.25\times10^{-3}$

$\delta d = d \times (0.25\times 10^{-3})$

= 0.0125 mm

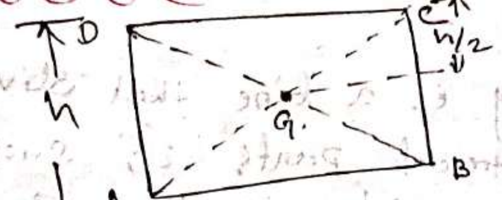
GEOMETRICAL PROPERTIES OF SECTIONS

Centroid :- The Plane figures (like traingle, circle, quadrilateral etc.) have only areas, but no mass. The centre of area of such figures is known as centroid.

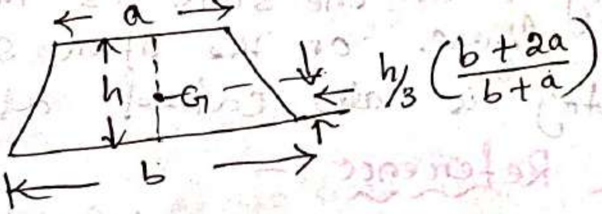
Centre of Gravity :- Centre of gravity of a body is a Point fixed in Position with respect to the body, through which the resultant of the weights of all constituent Particles of the body always Passes, no matter how the body is Placed. It is written as C.G.

C.G. by Geometrical considerations :-

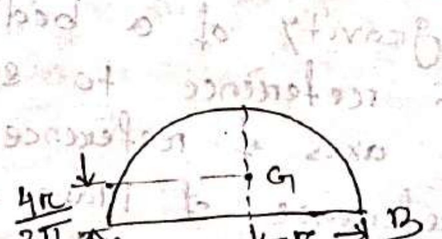
→ Rectangle :-



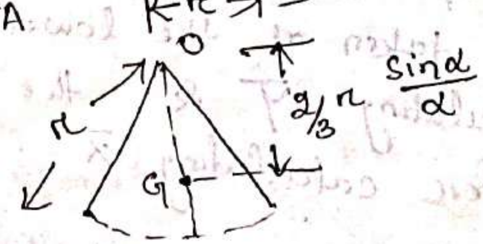
→ Trapezium :-



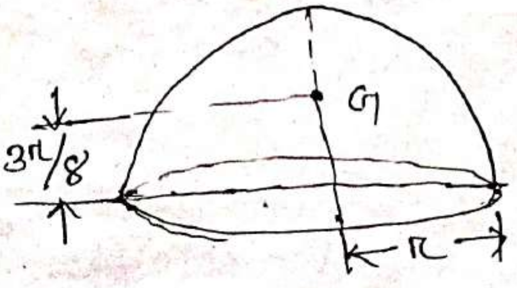
→ Semicircle :-



→ Circular Sector :-



→ Hemisphere :-



Symmetrical :- It means similar parts. i.e. one side is the same as the other. If you can draw a line down the centre of something & get two similar halves. It is known as symmetrical.

Asymmetrical :-

It means the opposite : the two sides are different in some way.
Asymmetrical things are irregular & crooked & don't match up perfectly when folded in half.

Axis of symmetry :-

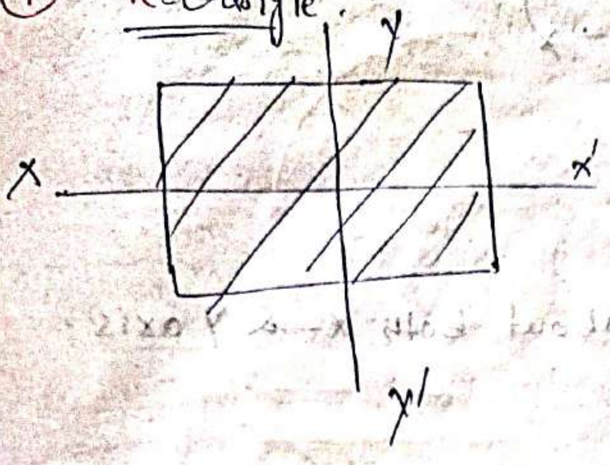
Axis of symmetry is a line that divides the figure in to two symmetrical parts in such a way that the figure on one side is the mirror image of the figure on the other side. Axis of symmetry is also called as line of symmetry.

Axis of Reference :-

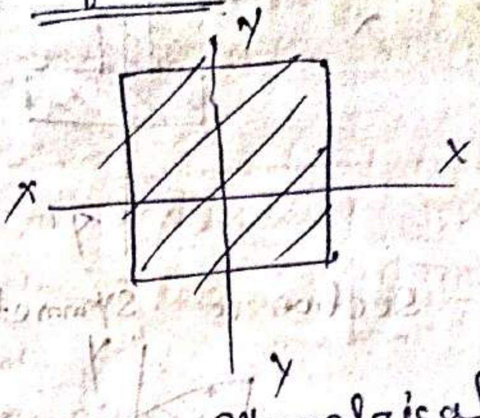
The centre of gravity of a body is always calculated with reference to some assumed axis known as axis of reference.
The axis of reference of plane figures is generally taken as the lowest line of the figure for calculating $\bar{y}$ & the left line of the figure for calculating $\bar{x}$.

Examples of symmetrical & unsymmetrical section.

① Rectangle :-

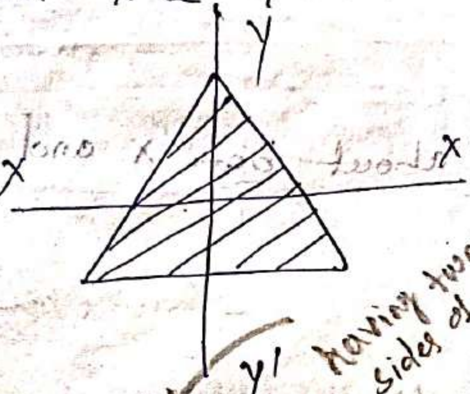


Square :-



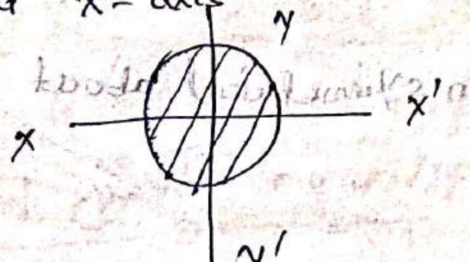
The rectangle & square sections are symmetrical about x & y- axis.

②



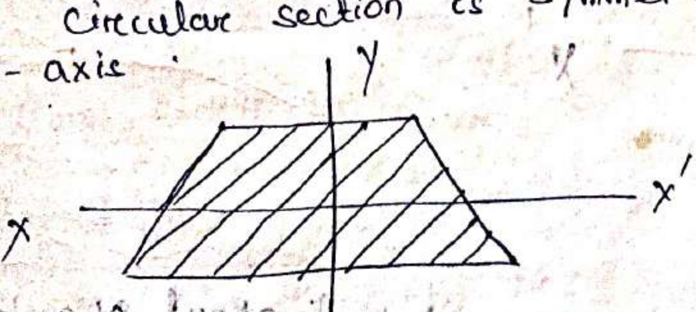
An isosceles traingle or equilateral traingle is symmetrical only about y- axis, but unsymmetrical about x- axis.

③



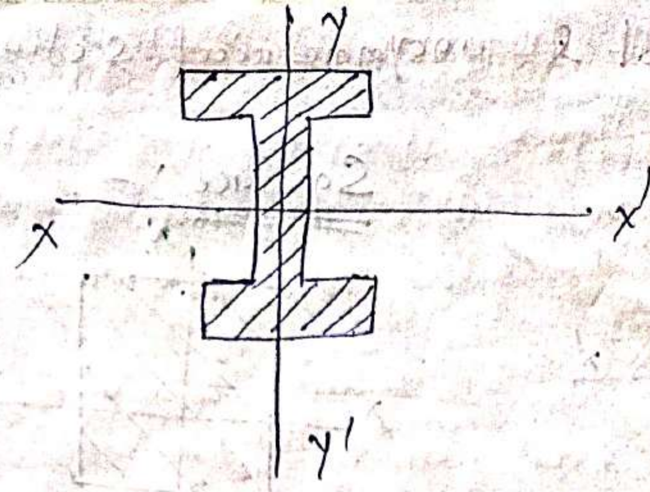
A circular section is symmetrical about both x & y- axis.

④



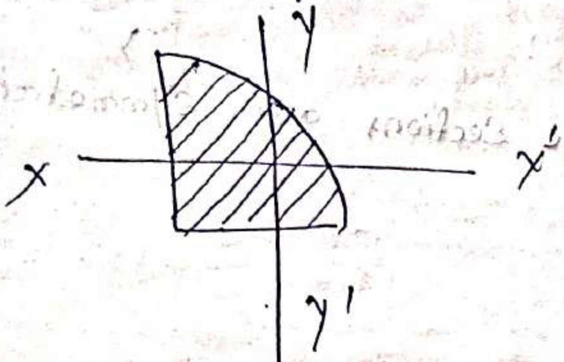
A trapezoidal section is symmetrical about y- axis but unsymmetrical about x- axis.

(5)



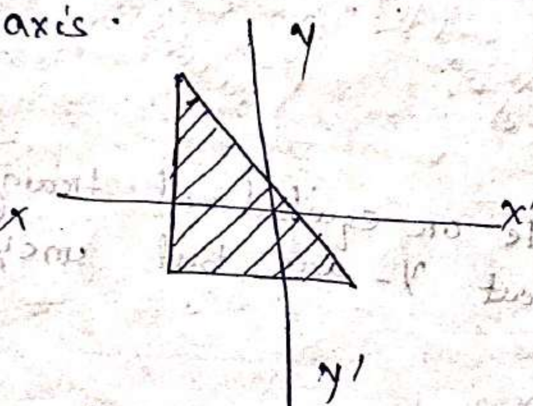
I - Section is symmetrical about both X - & Y axis.

(6)



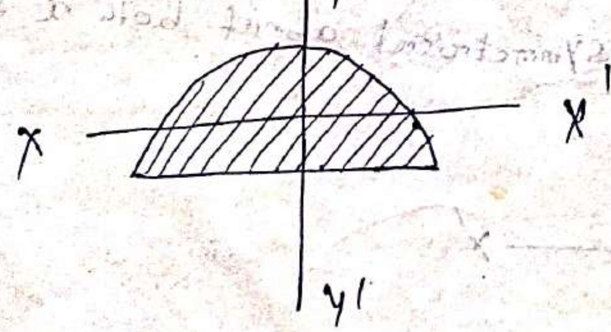
A quadrant is unsymmetrical about both X and Y - axis.

(7)



A right angle traingle is unsymmetrical about both X & Y - axis.

(8)

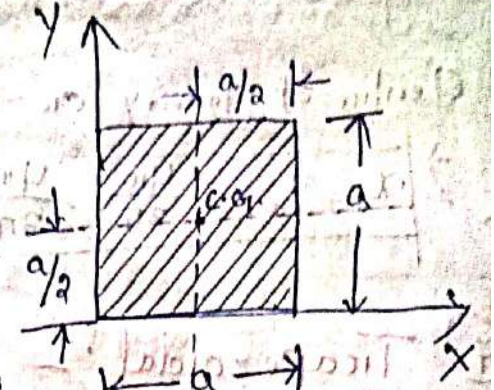


~~A Semi circle~~

A Semi circular section is symmetrical about Y axis & unsymmetrical about X - axis.

Centre of Gravity or centroid for different Geometric figures :-

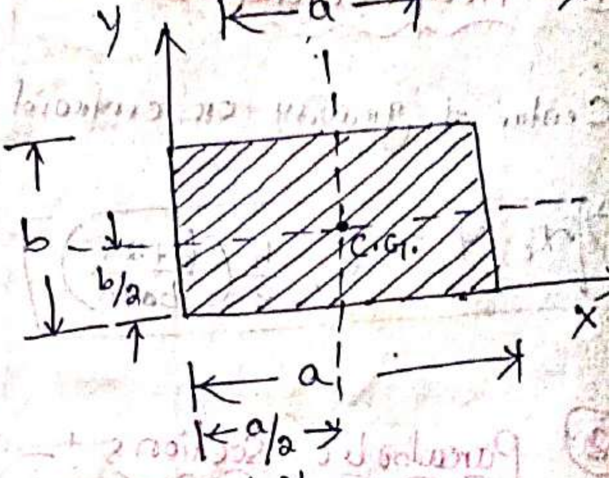
① Square :-



Centre of gravity or centroid =

$$(x_c, y_c) = (a/2, a/2)$$

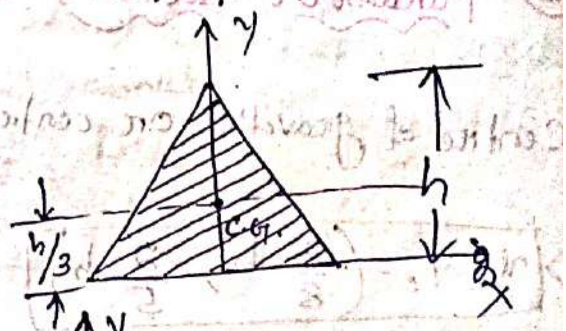
② Rectangle :-



Centre of gravity or centroid =

$$x_c, y_c = a/2, \; y_c = b/2$$

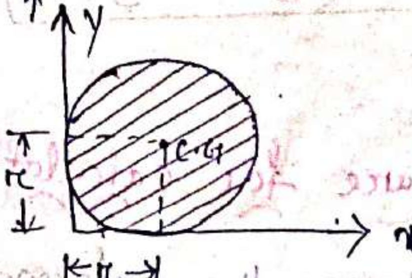
③ Triangle :-



Centre of gravity or centroid =

$$x_c, y_c = 0, h/3$$

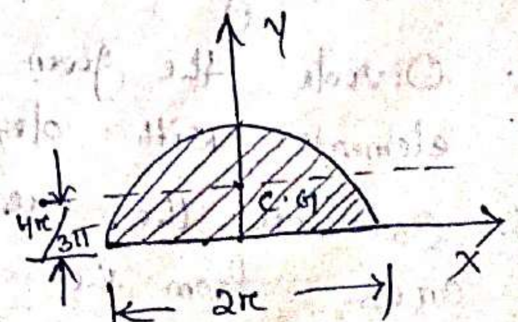
④ Circle :-



Centre of gravity or centroid

$$\Rightarrow (x_c, y_c) = (r, r)$$

⑤ Semi-circle :-



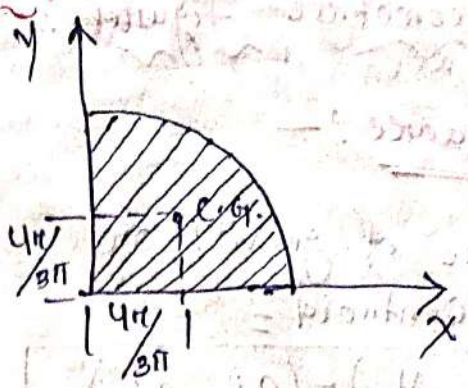
Centre of gravity or centroid

$$\Rightarrow x_c, y_c = 0, \frac{4r}{3\pi}$$

(6) Quarter Circle or Quadrant :-

Centre of gravity or centroid

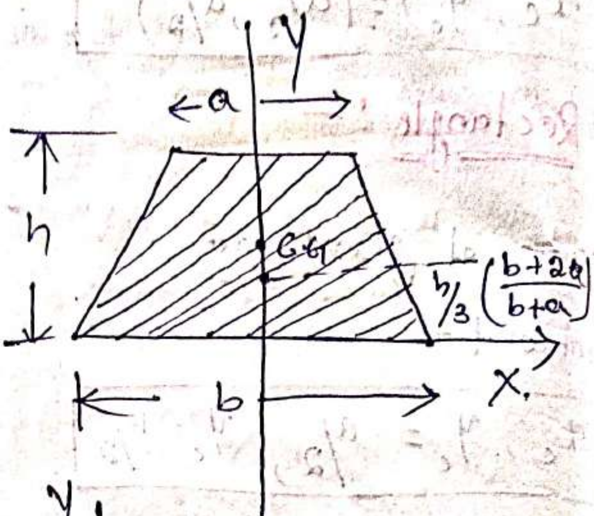
$$\Rightarrow x_c, y_c = \frac{4r}{3\pi}, \frac{4r}{3\pi}$$



(7) Trapezoidal :-

Centre of gravity or centroid

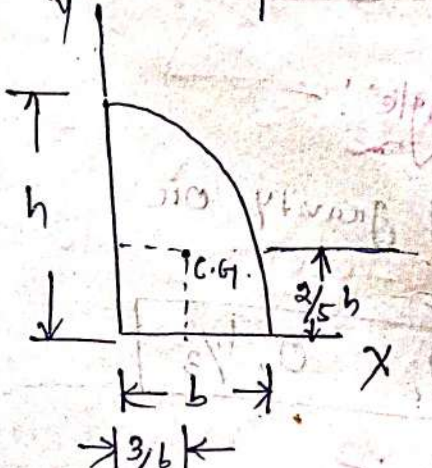
$$\Rightarrow x_c, y_c = 0, \frac{h}{3}\left(\frac{b+2a}{b+a}\right)$$



(8) Parabolic Sections :-

Centre of gravity or centroid

$$\Rightarrow x_c, y_c = \left(\frac{3}{8} b, \frac{2}{5} h\right)$$



Procedure for calculate the Position of C.G. :-

1. Choose the reference axis (if not given) by considering the axis of symmetry.
2. Divide the given area in to a number of elements with defined geometry.
3. Calculate the area & position of C.G. of each area from the reference axis.
4. Let $\bar{x}$ & $\bar{y}$ be the co-ordinates of the centre of gravity w.r.t. some axis of reference.

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3}$$

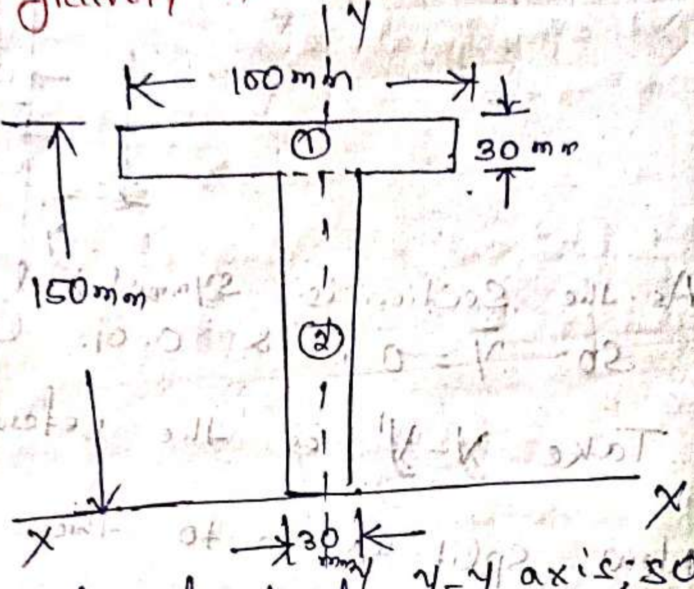
$$\bar{Y} = \frac{a_1 Y_1 + a_2 Y_2 + a_3 Y_3}{a_1 + a_2 + a_3}$$

5. If the figures symmetrical about x-axis then $\bar{Y} = 0$ & symmetrical about y-axis then $\bar{x} = 0$.

Problem-1

Find the centre of gravity of a 100 mm × 150 mm × 30 mm of T-section.

Sol^n:-



As the section is symmetrical about Y-Y axis, so $\bar{x} = 0$.
Therefore its centre of gravity lie on this axis.

Split up the section into two rectangles

Let Take reference axis is x-x axis

In Rectangle ①

Area $a_1 = 100 \times 30 = 3000\ mm^2$

distance $y_1 = (150 - 30/2) = 135\ mm$

Rectangle - ②

Area $a_2 = 120 \times 30 = 3600\ mm^2$

distance $y_2 = \frac{120}{2} = 60\ mm$

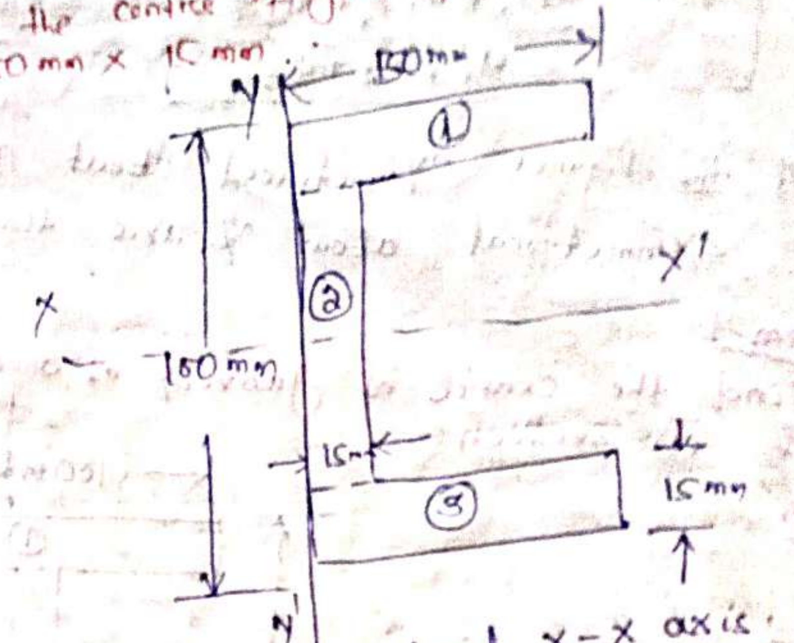
We know that distance of C.G. of the section from bottom.

$$\bar{Y} = \frac{a_1 Y_1 + a_2 Y_2}{a_1 + a_2} = \frac{(3000 \times 135) + (3600 \times 60)}{3000 + 3600}$$

$$= 94.1\ mm$$

Problem - 2 Find the centre of gravity of a channel section 150mm × 50 mm × 15 mm.

Soln:-



As the section is symmetrical about x-x axis. So $\bar{y} = 0$ & C.G. lie on this axis.

Take Y-Y' is the reference line.

Now split up in to three rectangle parts.

Rectangle - ①

Area $a_1 = 50 \times 15 = 750\ mm$

$x_1 = 50/2 = 25\ mm$

Rectangle - ②

Area $a_2 = $ ~~140 × 15~~ $70 \times 15 = 1050\ mm^2$

$x_2 = 15/2 = 7.5\ mm$

Rectangle - ③

Area $a_3 = 50 \times 15 = 750\ mm^2$

$x_3 = 50/2 = 25\ mm$

Centre of gravity of the section from the left part of the reference line.

$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3} = \frac{(750 \times 25) + (1050 \times 7.5) + (750 \times 25)}{750 + 1050 + 750}$$

$= 17.8\ mm$

Problem-3

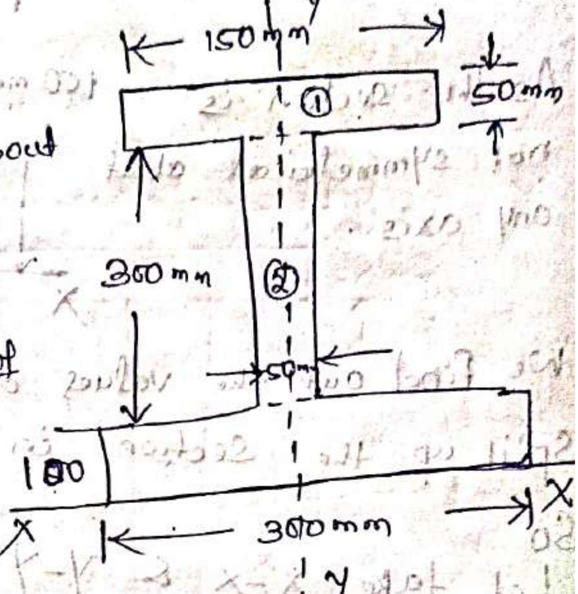
An I-section has the following dimensions in mm units.

Bottom flange = 300 × 100

Top flange = 150 × 50

Web = 300 × 50

Determine mathematically the position of centre of gravity of the section.



Soln:-

As the section is symmetrical about Y-Y axis. So $\bar{x} = 0$. So C.G. lie on this axis.

Let take X-X is the axis of reference

Now split up in to three rectangles.

Rectangle-①

Area $a_1 = 150 \times 50 = 7500\ mm^2$

$y_1 = 400 + 50/2 = 425\ mm$

Rectangle-②

Area $a_2 = 300 \times 50 = 15000\ mm^2$

$y_2 = 100 + 300/2 = 250\ mm$

Rectangle-③

Area $a_3 = 300 \times 100 = 30000\ mm^2$

$y_3 = 100/2 = 50\ mm$

Centre of gravity of the section from the bottom.

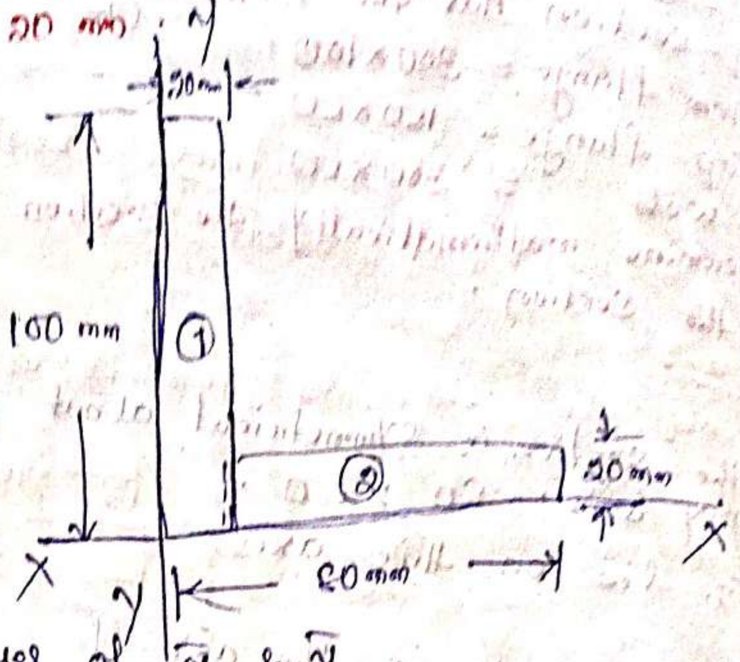
$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3} =$$

$$= \frac{(7500 \times 425) + (15000 \times 250) + (30000 \times 50)}{7500 + 15000 + 30000}$$

$= 160.7\ mm$.

Problem-4 Find the centroid of an unequal angle section 100 mm × 80 mm × 20 mm.

Sol^n:



As the section is not symmetrical about any axis.

We find out the values of $\bar{x}$ & $\bar{y}$.

Split up the section in to two rectangles

So

Let take X-X & Y-Y is the axis of reference.

Rectangle-①

$a_1 = 100 \times 20 = 2000\ mm^2$

$x_1 = 20/2 = 10\ mm$

$y_1 = 100/2 = 50\ mm$

Rectangle-②

$a_2 = 60 \times 20 = 1200\ mm^2$

$x_2 = 20 + 60/2 = 50\ mm$

$y_2 = 20/2 = 10\ mm$

$$\bar{X} = \frac{a_1x_1 + a_2x_2}{a_1 + a_2} = \frac{(2000 \times 10) + (1200 \times 50)}{2000 + 1200}$$

$$= 25\ mm$$

$$\bar{Y} = \frac{a_1y_1 + a_2y_2}{a_1 + a_2} = \frac{(2000 \times 50) + (1200 \times 10)}{2000 + 1200}$$

$$= 35\ mm.$$

CHAPTER-5

STRESSES IN BEAMS & SHAFTS

5.1 :- STRESSES IN BEAMS DUE TO BENDING :-

TOPICS TO BE COVERED :-

- ***BENDING STRESS IN BEAMS***
- ***THEORY OF SIMPLE BENDING***
- ***ASSUMPTIONS IN THE THEORY OF SIMPLE BENDING***
- ***MOMENT OF RESISTANCE***
- ***DERIVATION OF FLEXURE EQUATION***
- ***BENDING STRESS DISTRIBUTION- CURVATURE OF BEAM***
- ***POSITION OF NEUTRAL AXIS & CENTROIDAL AXIS***
- ***STIFFNESS EQUATION –FLEXURAL RIGIDITY-STRENGTH EQUATION-SIGNIFICANCE OF SECTION MODULUS-NUMERICAL PROBLEMS***

Bending stress :-

The bending moment at a section tends to bend or deflect the beam and the internal stresses resist its bending. The process of bending stops , when every cross-section sets up full resistance to the bending moment. The resistance offered by the internal stresses to the bending is called bending stress.

Assumptions in the theory of simple bending :-

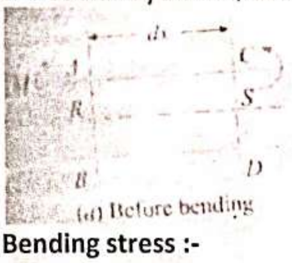
- The material of the beam is perfectly homogeneous
- The beam material is stressed within its elastic limit & thus obeys Hooke's law.
- The transverse sections, which were plane before bending , remains plane after bending also.
- Each layer of the beam is free to expand or contract , independently , of the layer above or below it.
- The value of E (young's modulus of elasticity) is the same in tension and compression.
- The beam is in equilibrium i.e. there is no resultant pull or push in the beam section.

Theory of simple bending :-

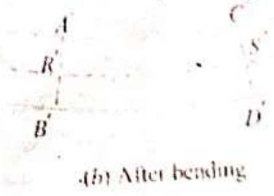
Consider a small length of a simply supported beam subjected to a bending moment .Now consider two section AB & CD which are normal to the axis of the beam RS.

Consider a small length of dx of the beam , there fore the curvature of the beam in this length, is taken to circular. The top layer of the beam has suffered compression and reduced to A'C'

The amount by which layer is compressed or stretched depends upon the position of layer with reference to RS. This layer RS which is neither compressed nor stretched is known as neutral plane or neutral layer. This theory of bending is called theory of simple bending.



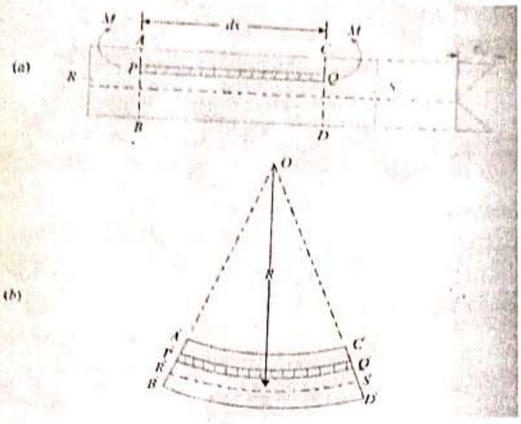
(a) Before bending



(b) After bending

Bending stress :-

Consider a small length dx of a beam subjected to a bending moment . Let this small length of beam bend in to an arc at a circle with 'O' as centre.



Let M= moment acting on the beam

θ = Angle subtended at the centre by the arc.

R= Radius of curvature of beam

For a layer PQ at a distance 'y' from the neutral axis of the beam. This layer be compressed after bending.

Change in length of this layer δl= PQ-P'Q'

Strain $\mathcal{E} = \delta l$/ original length

= PQ-P'Q'/PQ

Now from the geometry of the curved beam, we find the sections OP'Q' & OR'S' are similar

P'Q'/R'S' = R-y/R

=>1- (P'Q'/R'S') = 1-(R-y/R)

=>R'S'-P'Q'/ PQ = y/R

=>PQ-P'Q'/PQ = y/R (PQ= R'S' =Neutral axis)

$\mathcal{E}$ = y/R ($\mathcal{E}$= PQ-P'Q'/PQ)

The strain of a layer is proportional to its distance from the neutral axis .

We know that the bending stress σ_b = Strain × Elasticity = $\mathcal{E}$×E

= y/R × E = y × E/R

Since E & R are constants in this expression , therefore the stress at any point is directly proportional to y i.e. the distance of the point from the neutral axis.

The expression are σ_b/y = E/R

Or σ_b = E/R × y

Problem :- A steel wire of 5mm diameter is bent in to a circular shape of 5m radius. Determine the maximum stress induced in the wire. Take E =200Gpa.

Solution :-

Diameter of steel wire (d) = 5mm

Radius of circular shape (R) = 5m=5×10^3mm

Modulus of elasticity (E)=200 Gpa. =200×10^3 N/mm^2

We know that distance between the neutral axis of the wire & its extreme fibre

Y= d/2 = 5/2 = 2.5mm

& Maximum bending stress induced in the wire.

σ_b = E/R × y

$=\frac{200\times10^3}{5\times10^3}\times2.5 = 100$ N/mm^2 = 100Mpa (Ans)

Problem :-

A copper wire of 2mm diameter is required to be wound around a drum . Find the minimum radius of the drum., if the stress in the wire is not to exceed 80 Mpa. Take modulus of elasticity for the copper as 100 Gpa.

Solution :-

Diameter of wire (d) = 2mm

Maximum bending stress σ_b = 80 Mpa = 80 N/mm^2

Modulus of elasticity (E) = 100 Gpa = 100×10^3 N/mm^2

We know that distance between the neutral axis of the wire & its extreme fibre

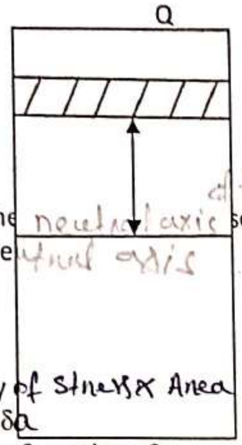
Y= 2/2 =1 mm

Minimum radius of the drum R= $y/\sigma_b \times E$

= $1/80 \times 100\times10^3$

= 1.25×10^3 mm =1.25m (Ans)

Position of neutral axis :-



Consider a section of the beam . Let the neutral axis of the section . Consider a small layer PQ of the beam section at a distance from the neutral axis

Let δa = Area of the layer PQ

The intensity of stress in the layer PQ

$\sigma = y\times E/R$

Total stress in the layer PQ = intensity of stress × Area

= $y \times E/R \times \delta a$

& Total stress of the section = $\Sigma y\times E/R \times \delta a = E/R\ \Sigma y.\delta a$

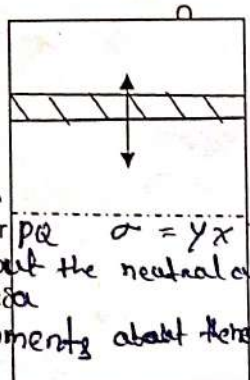
Since the section is in equilibrium , therefore total stress , from top to bottom, must be equal to zero.

$E/R\ \Sigma y.\delta a = 0$

Moment of resistance :-

Moment of resistance is that on one side of the neutral axis there are compressive stresses and on the other there are tensile stresses. This stresses form a couple which moment must be equal to the external moment . The moment of this couple which resist the external bending moment is known as the moment of resistance.

Consider a small layer PQ at a distance y from the neutral axis



Let δa = area of the layer PQ

The intensity of stress in the layer PQ $\sigma = y\times E/R$. Total stress in the layer PQ $= y\times E/R \times \delta a$

& moment of this total stress about the neutral axis

= $y \times E/R \times \delta a \times y = E/R \times y^2 \times \delta a$

The algebraic sum of al;l such moments about the neutral axis must be equal to M. Therefore

$M = \Sigma\ E/R \times y^2 \times \delta a = E/R\ \Sigma\ y^2 \times \delta a$

The expression $\Sigma\ y^2 \times \delta a$ represents the moment of inertia of the area of the whole section about the neutral axis .

Therefore M= E/R ×I (where I= moment of inertia)

M/I = E/R

We know that $\sigma/y = E/R$

$$\Rightarrow \frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

$\Rightarrow M/I = \sigma/y = E/R$

This the equation of theory of simple bending .

Modulus of section :-

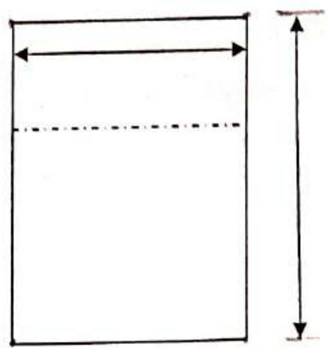
$M/I = E/R = \sigma/y$

$M/I = \sigma/y$

$\Rightarrow M = \sigma \times I/y$

$\Rightarrow M = \sigma \times Z$ ($I/y = Z$ i.e. section of modulus)

Rectangular section :- b



$I = bd^3/12$

$Y = d/2$

$Z = I/y$

$= bd^3/12 / d/2$

$= bd^3/12 \times 2/d$

$Z = bd^2/6$

Triangular section :-

$I = bd^3/36$

$Y = d/3$

$Z = I/Y$

$= bd^3/36/(d/3)$

$= bd^3/36 \times 3/d$

$\Rightarrow Z = bd^2/12$

Circular section :-

$I = \pi/64\ d^4$

$Y = d/2$

$Z = I/y$

$= \pi/64\ d^4 \times 2/d$

$= \pi d^3/32$

Strength of a section :-It is also termed as flexural strength of a section , which means the moment of resistance offered by it.

The relations $M/I = \sigma/y$ or $M = \sigma/y \times I$ & $M = \sigma Z$

__Problem :-__ A rectangular beam 60mm wide and 150mm deep is simply supported over a span of 6m . If the beam is subjected to central point load of 12 KN , find the maximum bending stress induced in the beam section .

__Solution :-__

Given data :-

Width (b) = 60mm

Depth (d) = 150mm

Span $l = 6\times10^3$ mm

& load W = 12 KN = 12×10^3N

We know that maximum bending moment at the centre of a simply supported beam subjected to a central point load

$M = Wl/4 = \frac{(12\times10^3)\times(6\times10^3)}{4} = 18 \times 10^6$ N.mm

& section modulus of the rectangular section

$Z = bd^2/6 = 60\times150^2/6 = 225\times10^3\ mm^3$

Maximum bending stress $\sigma_{max} = M/Z = 18\times10^6/(225\times10^3)$

$= 80\ N/mm^2 = 80$Mpa (Ans)

__Problem :-__A rectangular beam 300mm deep is simply supported over a span of 4metres. What uniformly distributed load the beam may carry , if the bending stress is not to exceed 120 Mpa. Take $I = 225 \times10^6\ mm^4$

__Solution :-__

Given data :-
Depth d= 300mm
Span l= 4m = 4×10^3mm
Maximum bending stress σ_{max} = 120 Mpa =120N/mm^2
Moment of inertia I= $225\times10^6 mm^4$
Let w= Uniformly distributed load the beam can carry.
We know that distance between the neutral axis of the section & extreme fibre
Y= d/2 = 300/2 = 150mm
& section modulus of the rectangular section Z= I/y = $225\times10^6/150$
= 1.5×10^6 mm^3

Moment of resistance M= $\sigma_{max} \times z$
= $120\times(1.5\times10^6) = 180\times10^6$N.mm
Maximum bending moment at the centre of a simply supported beam subjected to a uniformly distributed load (M)
$180\times10^6 = wl^2/8 = \frac{w\times(4\times10^3)^2}{8} = 2\times10^6\, w$
w=180/2 =90 N/mm = 90KN/m (Ans)

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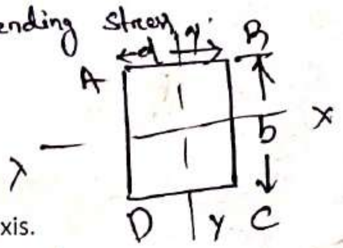
CHAPTER -5.4

COMBINED BENDING AND DIRECT STRESSES

TOPICS TO BE COVERED :-

- ***DIRECT AND INDIRECT STRESS***
- ***ECCENTRIC LOADS ON COLUMNS***
- ***COMBINED DIRECT & BENDING STRESS***
- ***MAXIMUM & MINIMUM STRESSES IN SECTION***
- ***PROBLEMS***
- ***LIMIT OF ECCENTRICITY***
- ***MIDDLE THIRD RULE OF SQUARE , RECTANGULAR, CIRCULAR SECTION***

Direct stress :-
A force of resistance offered by a body against before motion is known as direct stress.
$\sigma_0 = P/A$
Bending stress :- A force of resistance offered by the internal stress against bending is called bending stress.
$\sigma_b = M/I \times y$
Eccentric load :-A load whose line of action does not coincide to the axis of a column is known as eccentric load.
~~Resistance load~~ :- Resultant Stress = Direct stress ± Bending Stress
~~$\sigma_0 + \sigma_b$~~ Bending Stress



$\sigma_b = M/I \times y$
$\Rightarrow \sigma_b = p \times e/(db^3/12) \times (b/2)$
$\Rightarrow \sigma_b = p \times e \times 12/(db^3) \times b/2$
$\Rightarrow \sigma_b = 6pe/db^2$
Consider a column ABCD is subjected to an eccentric load about one axis.
P= eccentric load acting on the column
e=eccentric of the load

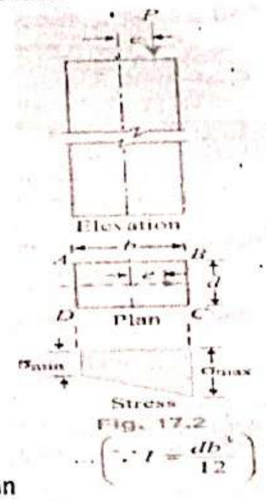


Fig. 17.2

$\left(\because I = \frac{db^3}{12}\right)$

b= base of column
d= thickness of the column
Area of column A =bXd
Direct stress $\sigma_0 = P/A$
Bending stress $\Rightarrow \sigma_b = 6pe/db^2$

Resultant stress = $\sigma_0 \pm \sigma_b$

$= P/A + 6pe/db^2$

$= \frac{P}{bXd} + \frac{6pe}{db^2}$

$= P/bd\,(1 \pm 6e/b)$

$\sigma_{max} = P/A\,(1+6e/b)$

$\sigma_{min} = P/A\,(1-6e/b)$

Problem :-A rectangular strut 150mm wide & 120mm thick. It carries a load of 180 KN at an eccentricity of 10mm in a plane bisecting the thickness . Find the maximum & minimum intensities of stress in the section.

Solution :-

Given data :-

$P = 180KN = 180\times10^3 N$

$e = 10mm$

$b = 150mm$

$d = 120mm$

Area = $bXd = 150\,X120 = 18X10^3 mm^2$

Maximum stress $\sigma_{max} = P/A\,(1+6e/b)$

$= \frac{180\times10^3}{18\times10^3}\,(1+6X10/150)$

$= 10(1+0.4) = 140Mpa$

$\sigma_{min} = P/A\,(1-6e/b)$

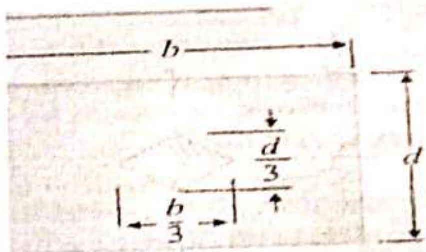
$= \frac{180\times10^3}{18\times10^3}\,(1-\,6X10/150)$

$= 10(1-0.4) = 6Mpa$

Middle third rule for column having rectangular section :-

σ_{max} is +ve & compressive by nature avoiding tension in a column

σ_{min} should be +ve & $\sigma_0 > \sigma_b$



$\sigma_{min} = P/A\,(1-6e/b) \geq 0 \;\Rightarrow\; \frac{P}{A}\left(1-\frac{6e}{b}\right) \geq 0$

$\Rightarrow 1-6e/b \geq 0$

$= 1 \geq 6e/b$

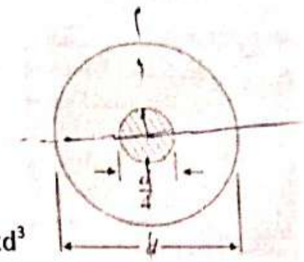
$\Rightarrow e \leq b/6$

The above result so that the eccentricity must be less than equal to b/6. Hence the greatest eccentricity of the load is b/6from the axis y-y axis if the load is applied any distance b/6 on any side of the axis y-y . The stress are wholly compressive &there is tensile stress . Similarly $e \leq b/6$ for applied eccentricity load with respect to axis x-x.

Middle quarter or fourth rule for column having circular section :-

$\sigma_0 = P/A = p/\pi/4\,X\,d^2 = 4p/\pi d^2$

$\sigma_b = M/I\,X\,y = pX\,e\,/\pi/64\,Xd^4\,X\,d/2$



$= 32pe/\pi d^3$

Resultant stress = $\sigma_0 \pm \sigma_b$

$=4p/\pi d^2 \pm 32\ pe/\pi d^3$

$= 4p/\pi d^2\ (1+8e/d)$

$= P/\pi/4d^2\ (1+8e/d)$

$= P/A\ (1+8e/d)$

For avoiding tension in a circular column section :-

$\sigma_{min} \geq 0$

$P/A\ (1-8e/d) \geq 0$

$\Rightarrow 1-8e/d \geq 0$

$\Rightarrow 1 \geq 8e/d$

$\Rightarrow e=d/8$

The above result so that the eccentricity must be less than d/8

If the load is applied any where within the circle of diameter d/4 the stress will be mainly wholly compressive.

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