

# **CIRCUIT AND NETWORK THEORY**

## **[TH-2]**

### **3<sup>RD</sup> SEM ELECTRICAL ENGG.**

*Under SCTE&VT, Odisha*

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## MODULE- I (10 hrs)

### 1.Magnetic coupled circuits. (Lecture -1)

#### 1.1.Self inductance

When current changes in a circuit, the magnetic flux linking the same circuit changes and e.m.f is induced in the circuit. This is due to the self inductance, denoted by L.

$$V = L \frac{di}{dt}$$

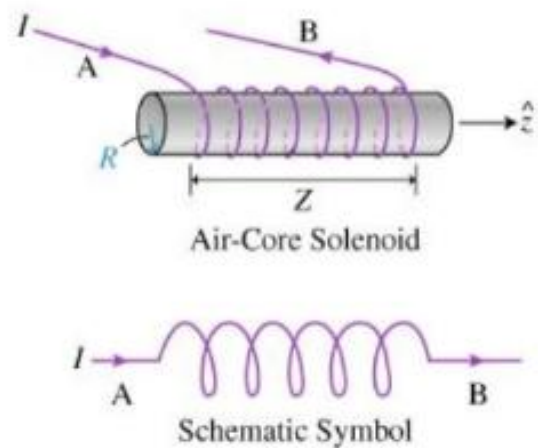
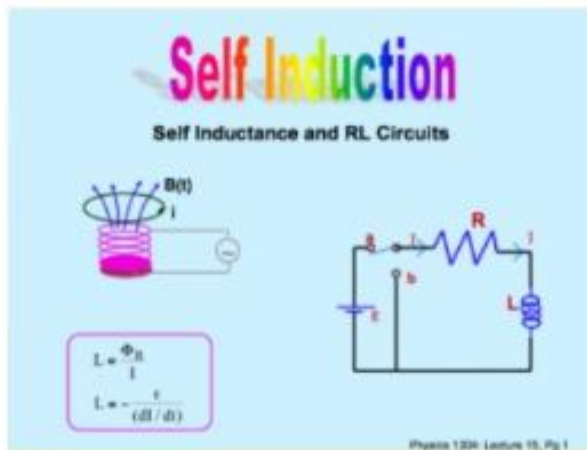


FIG.1

#### 1.2.Mutual Inductance

The total magnetic flux linkage in a linear inductor made of a coil is proportional to the current passing through it; that is,

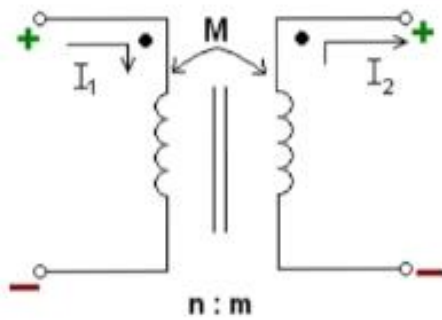


Fig. 2

$$\lambda = Li$$

. By Faraday's law, the voltage across the inductor is equal to the time derivative of the total influx linkage; given by,

$$L \frac{di}{dt} = N \frac{d\phi}{dt}$$

### 1.3. Coupling Coefficient

A coil containing N turns with magnetic flux  $\phi$  linking each turn has total magnetic flux linkage  $\lambda = N\phi$

. By Faraday's law, the induced emf (voltage) in the coil is

$$e = - \left( \frac{d\lambda}{dt} \right) = -N \left( \frac{d\phi}{dt} \right)$$

. A negative sign is frequently included in this equation to signal that the voltage polarity is established according to Lenz's law. By definition of self-inductance this voltage is also given by  $L di = dt \Phi$ ; hence,

The unit of flux ( $\phi$ ) being the weber, where  $1 \text{ Wb} = 1 \text{ V s}$ , it follows from the above relation that  $1 \text{ H} = 1 \text{ Wb/A}$ . Throughout this book it has been assumed that  $\phi$  and  $i$  are proportional to each other, making

$$L = (N\phi) / I = \text{constant}$$

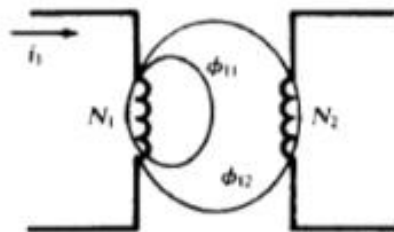


Fig.3

In Fig.3 , the total flux resulting from current  $i_1$  through the turns  $N_1$  consists of leakage flux,

$\phi_{12}$ , and coupling or linking flux,  $\phi_{21}$ . The induced emf in the coupled coil is given by  $N_2(d\phi_{12}/dt)$ . This same voltage can be written using the mutual inductance M:

$$M = N_1 \frac{d\phi_{21}}{di_2}$$

$$e = M \frac{di_1}{dt} = N_2 \frac{d\phi_{12}}{dt}$$

or

$$M = N_2 \frac{d\phi_{12}}{di_1}$$

Also, as the coupling is bilateral,

$$M = N_1 \frac{d\phi_{21}}{di_2}$$

$$M^2 = \left( N_2 \frac{d\phi_{12}}{dt} \right) \left( N_1 \frac{d\phi_{21}}{di_2} \right)$$

$$= \left( N_2 \frac{(k\phi_1)}{di_1} \right) \left( N_1 \frac{(k\phi_2)}{di_2} \right)$$

$$= k^2 \left( N_1 \frac{d\phi_1}{di_1} \right) \left( N_2 \frac{d\phi_2}{di_2} \right)$$

$$= k^2 L_1 L_2$$

Hence, mutual inductance, M is given by

$$M = k \sqrt{L_1 L_2}$$

And the mutual reactance  $X_M$  is given by

$$X_M = k \sqrt{X_1 X_2}$$

The coupling coefficient, k, is defined as the ratio of linking flux to total flux:

$$k = \frac{\phi_{12}}{\phi_1} = \frac{\phi_{21}}{\phi_2}$$

#### 1.4. Series connection of coupled circuit ( lecture 2)

When inductors are connected together in series so that the magnetic field of one links with the other, the effect of mutual inductance either increases or decreases the total inductance depending upon the amount of magnetic coupling. The effect of this mutual inductance depends upon the distance apart of the coils and their orientation to each other.

Mutually connected inductors in series can be classed as either "Aiding" or "Opposing" the total inductance. If the magnetic flux produced by the current flows through the coils in the same direction then the coils are said to be **Cumulatively Coupled**. If the current flows through the coils in opposite directions then the coils are said to be **Differentially Coupled** as shown below.

##### 1.4.1. Cumulatively Coupled Series Inductors

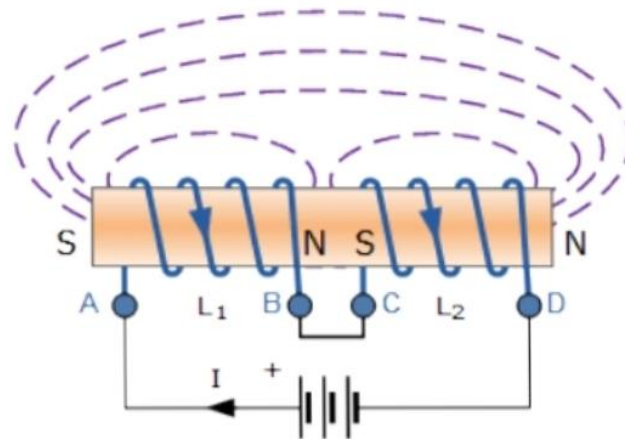


Fig.4

While the current flowing between points A and D through the two cumulatively coupled coils is in the same direction, the equation above for the voltage drops across each of the coils needs to be modified to take into account the interaction between the two coils due to the effect of mutual inductance. The self inductance of each individual coil,  $L_1$  and  $L_2$  respectively will be the same as before but with the addition of  $M$  denoting the mutual inductance.

Then the total emf induced into the cumulatively coupled coils is given as:

$$L_T \frac{di}{dt} = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + 2 \left( M \frac{di}{dt} \right)$$

Where:  $2M$  represents the influence of coil  $L_1$  on  $L_2$  and likewise coil  $L_2$  on  $L_1$ .

By dividing through the above equation by  $di/dt$  we can reduce it to give a final expression for calculating the total inductance of a circuit when the inductors are cumulatively connected and this is given as:

$$L_{\text{total}} = L_1 + L_2 + 2M$$

If one of the coils is reversed so that the same current flows through each coil but in opposite directions, the mutual inductance,  $M$  that exists between the two coils will have a cancelling effect on each coil as shown below.

#### 1.4.2. Differentially Coupled Series Inductors