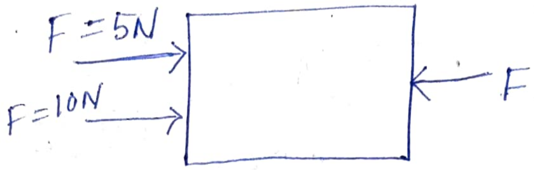
Scalar and vector

Physical quantity - The quantity which we can measure for that we need two thing Magnitude and direction.

→ some physical quantity have magnitude to represent but some physical quantity refers to both magnitude and direction.



here 5N or 10N represents a magnitude of physical quantity force

Physical quantity

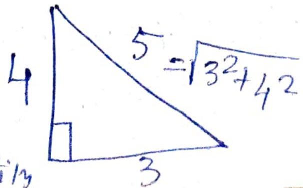
- Scalar (Scalars only magnitude)
  - Mass, volume, density, time
- Vector (Magnitudes also well as direction)
  - Force, displacement, velocity, acceleration

Scalar :- The physical quantity which can obey Algebraic Addition is known as Scalar quantity

Add :- 5kg + 3kg = 8kg (Algebric Add^n)

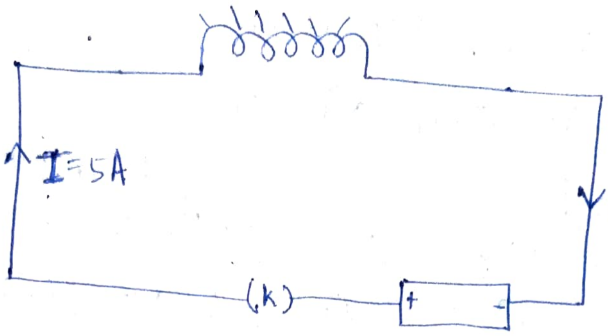
Vector Addition

The physical quantity which can obey Geometric addition known as vector quantity

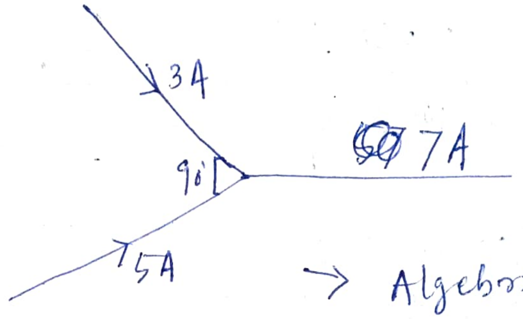


Magnitude :- Magnitude refers to a quantity or ~~[illegible]~~ capacity or size. (e.g.- your father ship)

Ex Current



both magnitude and direction



→ Algebric sum.

→ Current is that physical quantity which have both magnitude and direction but it obey Algebric sum. So we called it Tensor.

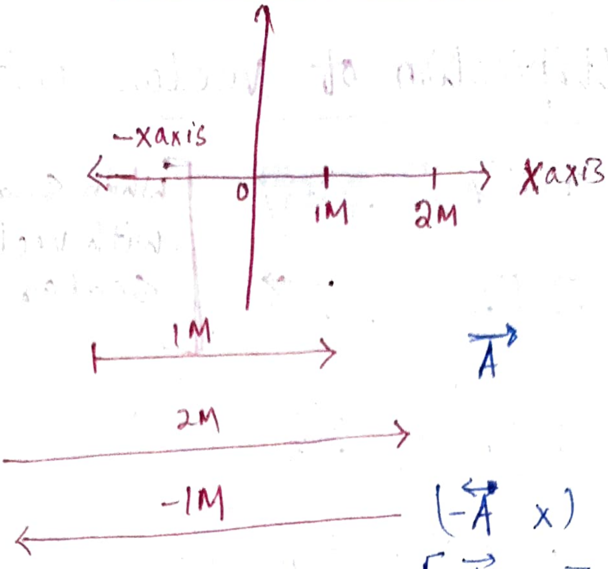
Tensor :- It is a scalar quantity which has direction.

Ex- Current, Pressure

Graphical representation of a vector :-

→ There is a sign to represent any physical quantity like $5^{\circ}C$ → Temp. (vector →) represents.

→ In vector (+ or -) represents their direction not their magnitude



$(-\overleftarrow{A}$ ✗$)$

$[-\vec{A}$ ✓$]$

$A \rightarrow$ Scalar quantity

$\vec{A} \rightarrow$ vector quantity

Problem-1

Draw the graphical representation of following physical quantity.

(i) $-\vec{A}$

(ii) $+2\vec{A}$

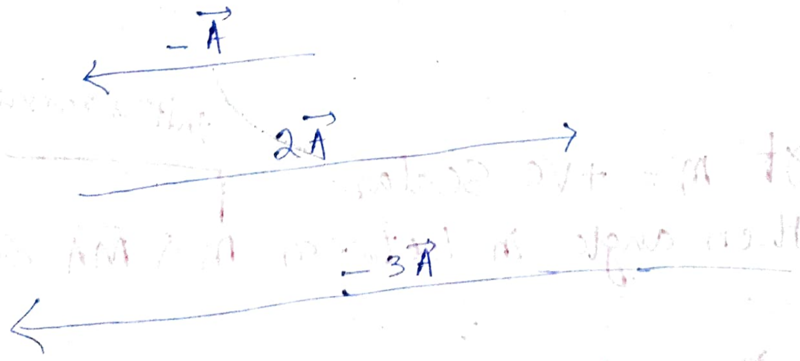
(iii) $-3\vec{A}$

Sol^n

(i)

(ii)

(iii)



Multipication of Vector with a Scalar

$m(\vec{A}) = m\vec{A}$

[When Scalar multiply with vector it gives scalar]

ex-1 $2(\vec{A}) = 2\vec{A}$

② $-2(\vec{A}) = -2\vec{A}$

③ $0(\vec{A}) = \vec{0 \cdot A} = 0$

EX-1

$+\vec{A}$

$m = +2$

$m(\vec{A}) = +2\vec{A}$

[+ve scalar multiply with a vector then angle 0°]

EX-2

$m - 2$

$m(\vec{A}) = -2\vec{A}$

[-ve scalar multiply with +ve vector then angle is 180°]

Note

i) If m = +ve Scalar
then angle in between m & $m\vec{A}$ is 0°

initial resultant

ii) If m = -ve Scalar
then angle in between m & $m\vec{A}$ is 180°

Task

(i) If $m = +3$ then Draw $m\vec{A}$

(ii) If $m = -3$ then draw $m\vec{A}$

Types of vector

(i) Zero vector / Null vector $(\vec{0})$

(ii) Unit vector

(iii) orthogonal vector / Base vector

(iv) Equal vector

(v) Negative vector

(vi) Co-linear vector

(vii) Co-planer vector

(viii) Localised vector

(ix) Non-localised vector

(i) Zero vector $(\vec{0})$:-

1) Magnitude $= 0$

2) It has no specific direction

Properties :-

(1) $\vec{A} + \vec{0} = \vec{A}$ (no change in magnitude & direc)

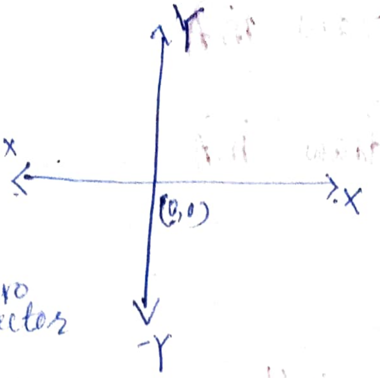
(2) $\vec{A} - \vec{0} = \vec{A}$

(3) $\vec{A} \times \vec{0} = \vec{0}$

(4) $\vec{A} \cdot \vec{0} = 0$

$\vec{A} - \vec{A} = \vec{0}$

EX



arbitrary dirch → zero vector

(ii) Unit vector :-

$\vec{A} = A\hat{A}$

magnitude

Let's a student ask if $\vec{A}$ is a vector how to represent its magnitude & direction.

Though only magnitude means scalar we don't put cap over A.

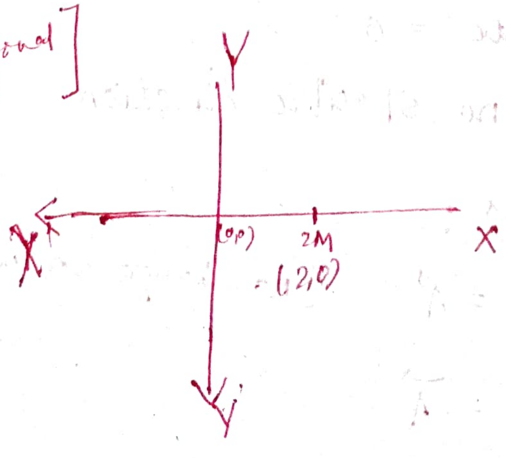
(i) $\hat{A}$ is a unit vector whose magnitude = 1

(ii) It's direction is along the direction of its parent vector.

ex $\vec{B} = B \cdot \hat{B}$ ($\hat{B}$ directed towards $\vec{B}$)

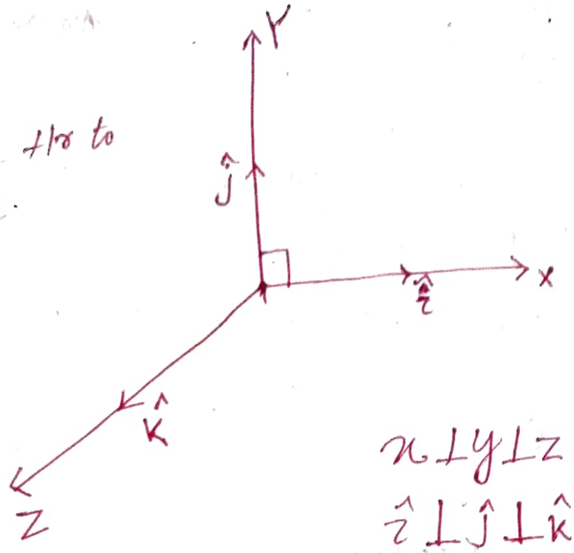
ex $\vec{C} = C \cdot \hat{C}$

Question [ORthogonal vector]



(iii) Orthogonal vector / Base vector :-

x, y, z are ⊥r to each other.



$x \perp y \perp z$

$\hat{i} \perp \hat{j} \perp \hat{k}$

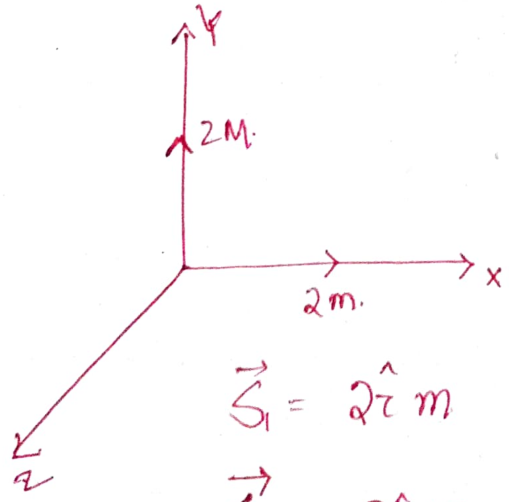
The unit vector which are ⊥r with each other is known as orthogonal vector.

ex- $\hat{i}, \hat{j}, \hat{k}$

→ All orthogonal vector bound to be unit vector

→ Unit vector could be orthogonal or not be orthogonal

EX



$\vec{S_1} = 2\hat{i}\ m$

$\vec{S_2} = 2\hat{j}\ m.$

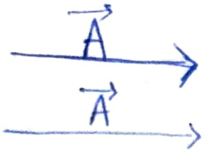
$\vec{S_3} = 2\hat{k}\ m$

$\vec{S} = -2\hat{i}\ m$

magnitude = 2m

direction = -xa

iv) Equal vector :-



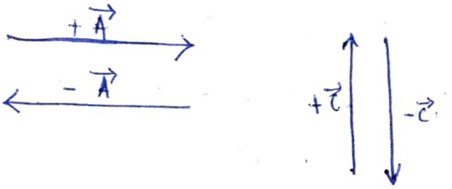
Length represent → magnitude
Arrow represent direction

vector having same magnitude and same direction is known as Equal vector

→ Angle betn them is zero (0°)

$$+\vec{A} = +\vec{C}$$

v) Negative vector :-

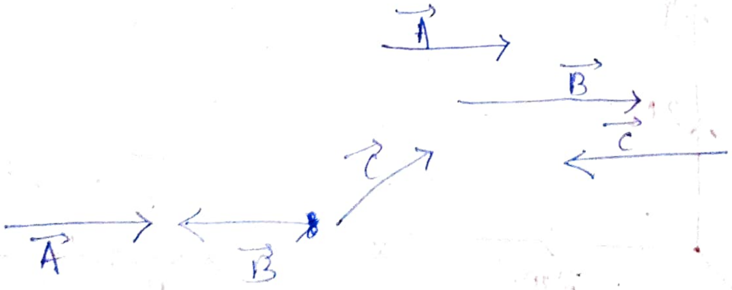


Same in magnitude but opposite in direction is known as Negative vector.

vi) Co linear vector :-

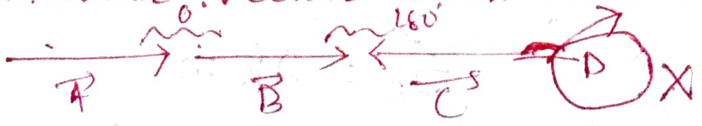
↗ same

The vector having same line of action is called Co-linear vector.



here $\vec{A}$ & $\vec{B}$ are co linear but $\vec{c}$ is not as the line of action of $\vec{c}$ is not same

here condition
→ For colinear vector Must be 0° or 180°



→ If the angle between two vectors is not 0° or 180° then it is called Non. co-linear vector

ex-

Colinear vector

Parallel vector

$\theta = 0^\circ$ (Angle betn vectors)

Magnitude may or may not be equal.

Anti parallel vector

$\theta = 180^\circ$

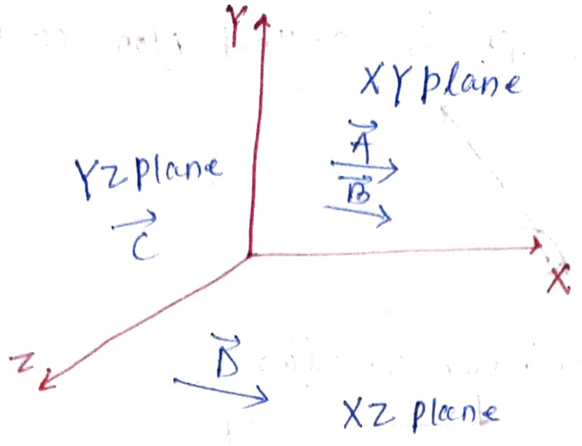
⊛ What is the difference between equal and parallel vectors ?

Ans In case of equal vectors magnitude ~~may~~ should be same.
but in case of parallel magnitude may be or may not be same.

→ All equal vectors are parallel but may or mayn't be parallel vectors are equal.

⇒ Incase of Anti parallel vector direction must be opposite magnitude may or mayn't be same but incase of Negative vector magnitude must be same and direction must be opposite.

vii) Coplaner vector :



→ Here $\vec{A}$ & $\vec{B}$ are coplaner vector.

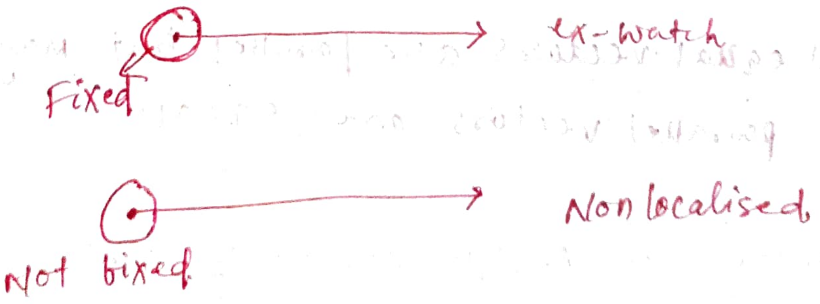
→ $\vec{A}$ & $\vec{C}$ or $\vec{C}$ & $\vec{D}$ can't be coplaner vector.

When two vectors lies on the same plane then it is said to be coplaner vector.

→ When ~~they are~~ vectors are not in same plane is called non-coplaner vector.

viii) Localised vector

Vector whose initial point is fixed is known as localised vector.



Home task

Write mathematical vector form

1. 2m along $-x$ axis.
2. 0.5m along z axis

Vector Addition

$$\vec{A} + \vec{B} = \vec{R}$$

(If we add two vector then the resultant must be a vector)

We have three methods for vector Addition.

(i) Triangle's law of vector Addition

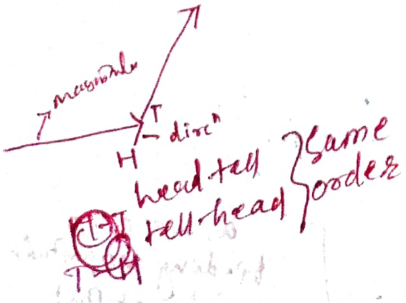
(ii) Parallelogram law of vector Addition

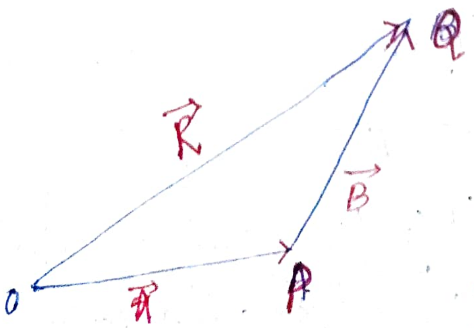
(iii) Polygons law of vector Addition

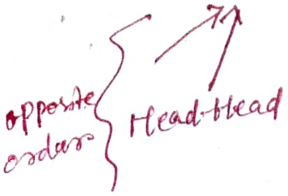
(i) Triangle's law

If two vectors are represents on the two sides of a triangle are taken in same order then their resultant vector must be represent on the third side of this triangle is taken in opposite order.

i.e. $\vec{A} + \vec{B} = \vec{R}$

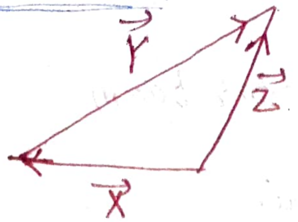






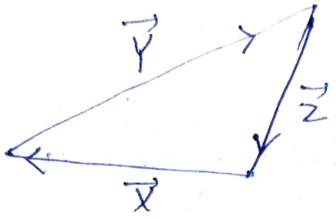
Graphical method

Ex



$\vec{Z} = \vec{X} + \vec{Y}$

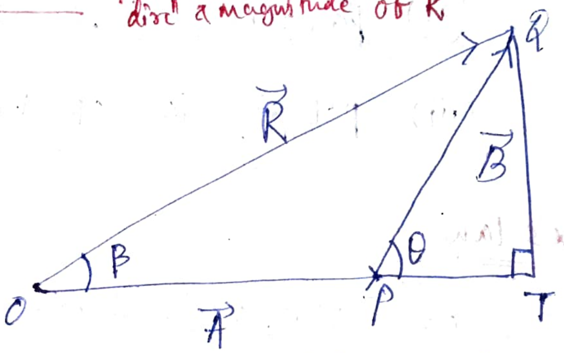
(Z is resultant vector as Z & y are opposite order but x & y same order.



$\vec{X} + \vec{Y} + \vec{Z} = 0$ (Though their resultant vector is zero so it is a null vector)

Analytical method

: In analytical method we will find 'dir' & magnitude of R



β = angle in between $\vec{R}$ and $\vec{A}$

θ = angle in between $\vec{A}$ and $\vec{P}$

[→ we will taken angle between two vectors we want to add]

β → represents the direction of resultant vector $\vec{R}$.

From Δ OQT

$\angle OTQ = 90^\circ$

According to Pythagoras $[h^2 = b^2 + p^2]$

$\Rightarrow OQ^2 = OT^2 + QT^2$

$\Rightarrow OQ^2 = (OP + PT)^2 + QT^2$

[as we are finding Magnitude so i donot put ~~vector~~ over any term.

$\Rightarrow OQ^2 = \underset{?}{OP^2 + PT^2} + 2 \cdot OP \cdot PT + \underset{?}{QT^2}$ —— (1)

Note [In ΔPTQ if h and angle(θ) known then to find base we use $\cos\theta$ to find perpendicular we use $\sin\theta$ we know]

Consider ΔPQT

$$\cos\theta = \frac{B}{H} = \frac{PT}{PQ} = \frac{PT}{B}$$

$$\Rightarrow \boxed{PT = B\cos\theta}$$

$$\sin\theta = \frac{P}{H} = \frac{QT}{PQ} = \frac{QT}{B}$$

$$\Rightarrow \boxed{QT = B\sin\theta}$$

Putting the values of PT & QT in eqn (1)

$$\Rightarrow R^2 = A^2 + B^2\cos^2\theta + 2AB\cos\theta + B^2\sin^2\theta$$

$$\Rightarrow R^2 = A^2 + B^2(\cos^2\theta + \sin^2\theta) + 2AB\cos\theta$$

$$\Rightarrow R^2 = A^2 + B^2 + 2AB\cos\theta$$

$$\Rightarrow \boxed{R = \sqrt{A^2 + B^2 + 2AB\cos\theta}}$$

θ = Angle between $\vec{A}$ & $\vec{B}$ or two vector.

→ Whenever it is asked find the ~~value~~ direction of resultant vector then we have to find the value of β.

For direction of resultant vector

Consider $\triangle OTQ$

$$\angle OTQ = 90^\circ$$

$$\tan\beta = \frac{P}{B} = \frac{QT}{OT} = \frac{QT}{OP+PT}$$

$$\Rightarrow \tan\beta = \frac{B\sin\theta}{A + B\cos\theta}$$

→ [I don't put vector sign over A & B Because we can add, subtract or multiply but we can't divide vector so i have not put it!]

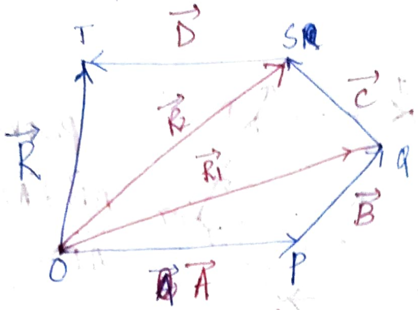
$$\Rightarrow \beta = \tan^{-1}\left[\frac{B\sin\theta}{A+B\cos\theta}\right]$$

β = direction of resultant vector.

(iii) Polygon's law :-

Statement - If no. of vectors are acting simultaneously act at a point then the vectors are represent on the side of an open polygon are taken in same order then their resultant vector must be represent on the closing side of this polygon taken in opposite order.

$$\vec{A} + \vec{B} + \vec{C} + \vec{D} = \vec{R}$$



From ΔOPQ

$$\vec{R_1} = \vec{A} + \vec{B}$$

[As $\vec{A}$ and $\vec{B}$ are in same order but $\vec{R_1}$ in opposite order so it is a resultant vector]

From ΔOQS

$$\vec{R_2} = \vec{R_1} + \vec{C}$$

$$\vec{R_2} = (\vec{A} + \vec{B}) + \vec{C}$$

From ΔOST

$$\vec{R} = \vec{R_2} + \vec{D}$$

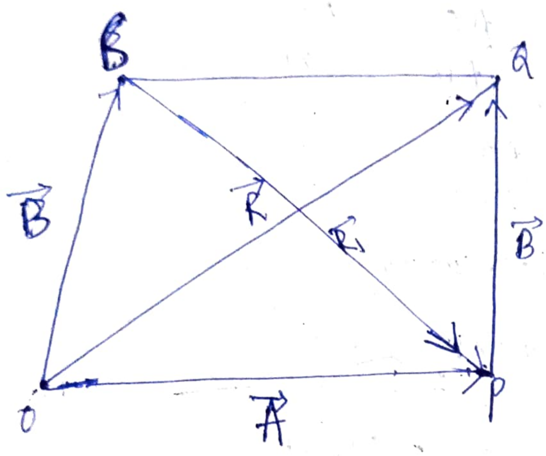
$$\rightarrow \boxed{\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}}$$

→ Polygon's law principle based upon triangles law

Parallelogram law of vector Addition :

Statement - If two vectors are represents on the two adjacent side of a parallelogram drawn from a common point then their resultant vector must be drawn from that point and it represent on the diagonal of this parallelogram

i.e $\vec{A} + \vec{B} = \vec{R}$



From ΔOPQ

$$\vec{OQ} = \vec{OP} + \vec{PQ}$$

$$\vec{R} = \vec{A} + \vec{B}$$

ΔOSP

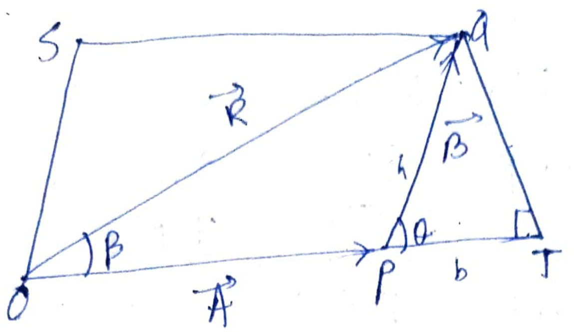
$$\vec{OP} = \vec{OS} + \vec{SP}$$

$$\Rightarrow \vec{A} = \vec{B} + \vec{R_1}$$

$$\Rightarrow \boxed{\vec{R_1} = \vec{A} - \vec{B}}$$

→ In Parallelogram law of vector Addition if one diagonal is vector Add^n then another one must be the vector Subtraction.

Analytical method : [we will find magnitude & dirⁿ]



[One perpendicular drawn QT from point Q]

FROM Δ OQT

$$OQ^2 = OT^2 + QT^2$$

$$OQ^2 = (OP+PT)^2 + QT^2$$

$$\Rightarrow OQ^2 = OP^2 + PT^2 + 2\,OP\,PT + QT^2 \quad \text{---} \ (1)$$

(→ not put bcz i find magnitude here)

FROM Δ PQT

$\angle PTQ = 90°$

$$\cos\theta = \frac{b}{h} = \frac{PT}{PQ} = \frac{PT}{B} \qquad \Big| \qquad \sin\theta = \frac{P}{h} = \frac{QT}{PQ} = \frac{QT}{B}$$

$$\Rightarrow QT = B\sin\theta$$

$$\Rightarrow PT = B\cos\theta$$

Put all these values in equation (i) becomes.

$$R^2 = A^2 + B^2\cos^2\theta + 2AB\cos\theta + B^2\sin^2\theta$$

$$\Rightarrow R^2 = A^2 + B^2(\underbrace{\cos^2\theta + \sin^2\theta}_{1}) + 2AB\cos\theta$$

$$\Rightarrow R^2 = A^2 + B^2 + 2AB\cos\theta$$

$$\Rightarrow R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

[θ = angle between 2 vectors]

Now we will find direction

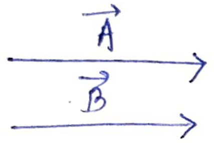
From ΔOQT,

$\angle OTQ = 90°$

$$\tan\beta = \frac{QT}{OT} = \frac{QT}{OP+PT}$$

$$\Rightarrow \tan\beta = \frac{B\sin\theta}{A+B\cos\theta}$$

Case-1 (Parallel vector)



$\theta = 0°$

Magnitude

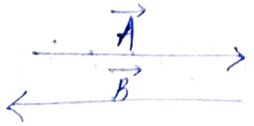
$$R = \sqrt{A^2+B^2+2AB\cos\theta}$$
$$= \sqrt{A^2+B^2+2AB}$$
$$= \sqrt{(A+B)^2}$$
$$= A+B$$

Direction

$$\tan\beta = \frac{B\sin\theta}{A+B\cos\theta}$$
$$= \frac{B\sin 0°}{A+B\cos 0°}$$
$$= \frac{0}{A+B} = 0$$
$$\tan\beta = \tan 0°$$
$$\beta = 0°$$

→ In this case ~~Magnitude and~~ direction is same as direction of two vector

Case-2 (Anti Parallel vector)



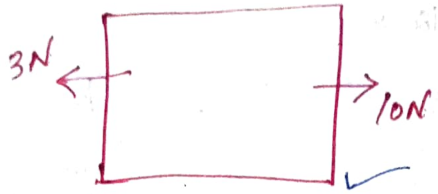
$\theta = 180°$

$$R = \sqrt{A^2+B^2+2AB\cos 180°}$$
$$= \sqrt{A^2+B^2-2AB}$$
$$= \sqrt{(A-B)^2}$$
$$= A-B$$

→ In case of Antiparallel vector we subtract both the magnitude. Direction is towards the vector having more/much magnitude.

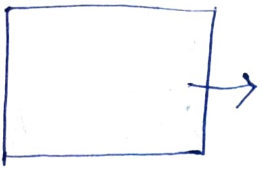
e.g

Find the net force.

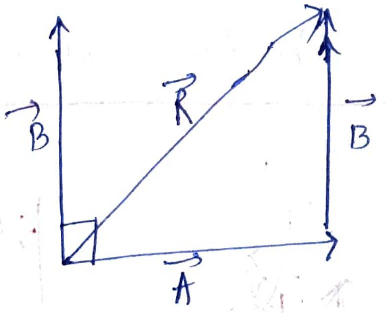


$= 10N - 3N$

$= 7N$ (∴ directed towards 10N)

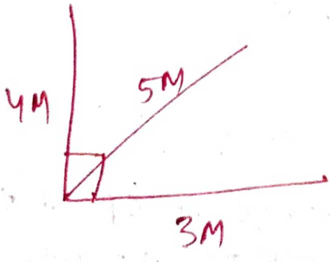


Case-3 Perpendicular vector



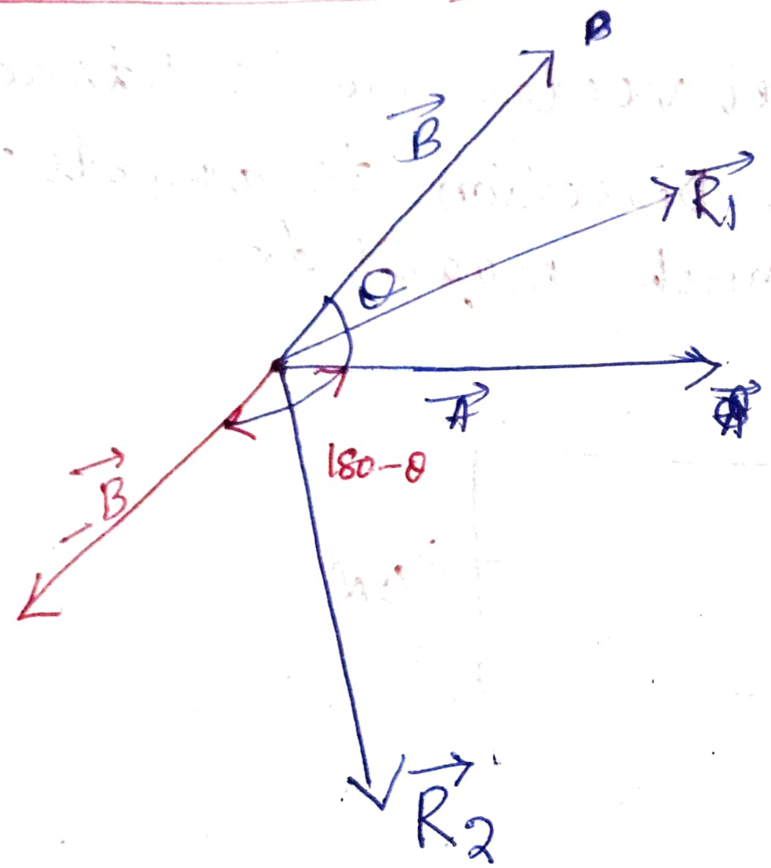
$$R = \sqrt{A^2 + B^2 + 2AB\cos 90^\circ}$$
$$= \sqrt{A^2 + B^2}$$

e.g



$\sqrt{3^2 + 4^2} = 5$

Vector ~~Addition~~ Subtraction



Here Angle = θ

$$\vec{R_1} = \vec{A} + \vec{B}$$

$$\therefore \vec{R_1} = \sqrt{A^2+B^2+2AB\cos\theta}$$

$$\vec{R_2} = \vec{A} + (-\vec{B})$$

$$\Rightarrow \vec{R_2} = \vec{A} - \vec{B}$$

Here Angle(θ) = 180 − θ

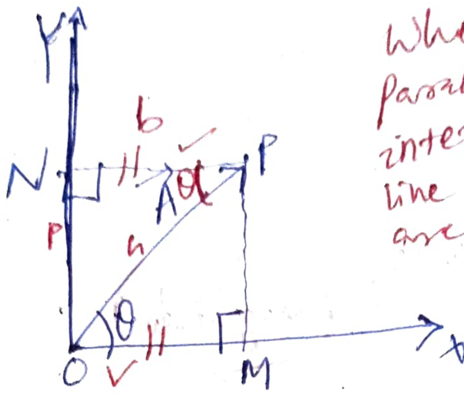
$$R_2 = \sqrt{A^2+B^2+2AB\cos(180-\theta)}$$

$$\boxed{R_2 = \sqrt{A^2+B^2-2AB\cos\theta}}$$

[$\cos(180-\theta) = -\cos\theta$]

Resolution of a vector :-

Spliting of a vector is called resolution of a vector



Whenever two parallel lines are intersect by third line the opposite angles are equal

In ΔOMP

$$\cos\theta = \frac{b}{h} = \frac{OM}{OP} = \frac{OM}{A}$$

[Sign not given over vector bcz we can't divide vector]

$\Rightarrow \boxed{OM = A\cos\theta}$ (For x-axis - $\cos\theta$)

In ΔOPN

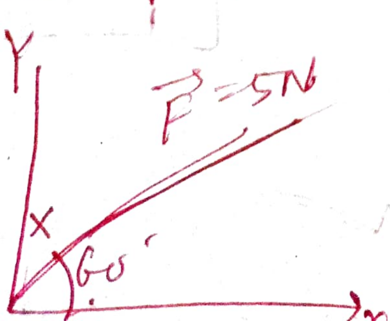
$$\sin\theta = \frac{P}{h} = \frac{ON}{OP} = \frac{ON}{A}$$

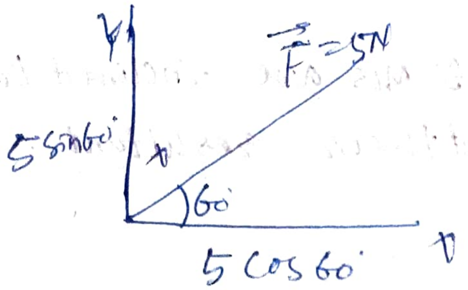
$\Rightarrow \boxed{ON = A\sin\theta}$ (For y-axis $\sin\theta$)

$A\cos\theta$ & $A\sin\theta$ are two component of $\vec{A}$

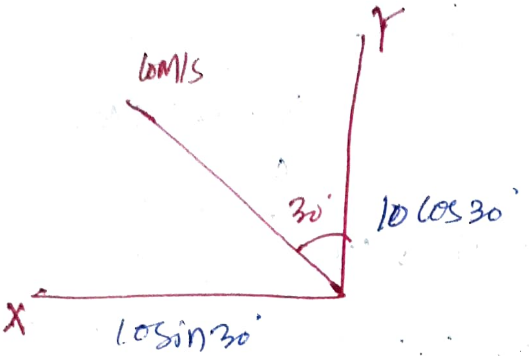
rectangular component

example





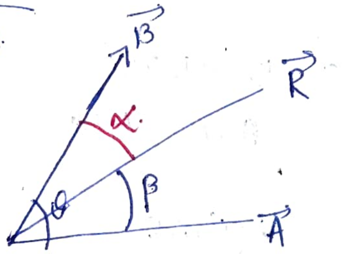
ex



Formula required for problems on scalar & vector

(i) $\vec{R} = \vec{A} + \vec{B}$

$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$



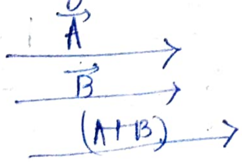
$\beta \rightarrow$ dirc^n indicates

$$\tan\beta = \frac{B\sin\theta}{A + B\cos\theta}$$

Suppose angle α taken then

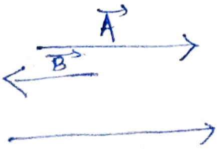
$$\tan\alpha = \frac{A\sin\theta}{B + A\cos\theta}$$

(ii) From parallelogram law of vector Add^n



→ Resultant (magnitude of two vector)

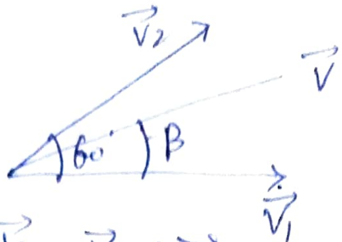
In Antiparallel vectors



In Antiparallel vector ~~resultant~~ two vectors subtracted to get Resultant and direc^n towards the vector having more magnitude

Q.1 Two velocities each 5 M/s are inclined to each other at 60° Find their resultant.

Sol:



$\vec{V} = \vec{V_1} + \vec{V_2}$

$V_1 = V_2 = 5 M/s$
$\theta = 60°$

Magnitude $V = \sqrt{V_1^2 + V_2^2 + 2V_1V_2\cos\theta}$

$= \sqrt{5^2 + 5^2 + 2\cdot5\cdot5\cdot\cos 60}$

$= \sqrt{5^2 + 5^2 + \cancel{2}\cdot5^2\cdot\frac{1}{\cancel{2}}}$

$= \sqrt{3\times5^2}$

$= 5\sqrt{3}$ M/s

$= 5\times1.732$
$= 8.660$ M/s

direction $\tan\beta = \frac{V_2\sin\theta}{V_1 + V_2\cos\theta} = \frac{5\sin60°}{5+5\cos60°}$

$= \frac{\cancel{5}(\sin60)}{\cancel{5}(1+\cos60°)}$

$= \frac{\sin60°}{1+\cos60°}$

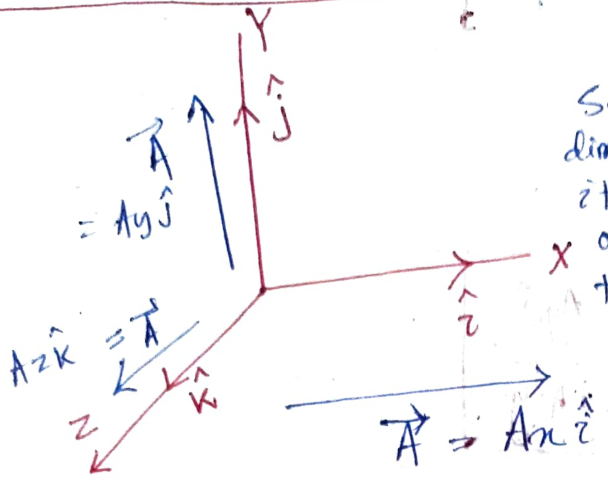
$\tan\beta = \frac{\frac{\sqrt{3}}{2}}{1+\frac{1}{2}} = \frac{\frac{\sqrt{3}}{2}}{\left(\frac{3}{2}\right)} = \frac{\sqrt{3}}{\cancel{2}}\times\frac{\cancel{2}}{3}$

$= \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan\beta = \tan30°$

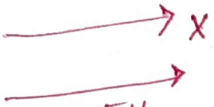
$\boxed{\beta = 30°}$

Vector in one dimension



A vector is said to be one dimensional vector if it is parallel with any one of axis of a three co-ordinate axis

e.g.

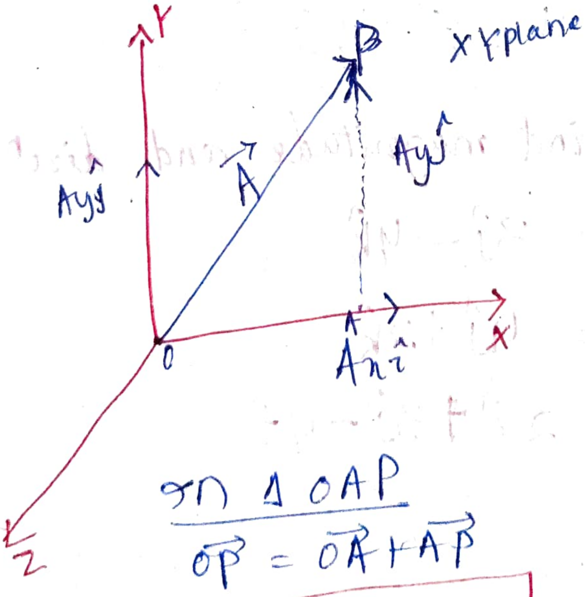


$F = 5N.$

$\vec{F} = 5\hat{i}$

Vector in two dimension

A vector is said to be two dimensional vector if it is lying in a two dimensional plane.



In Δ OAP

$\vec{OP} = \vec{OA} + \vec{AP}$

$\Rightarrow \boxed{\vec{A} = A_x\hat{i} + A_y\hat{j}}$

here $\vec{A}$ having two unit vector $\hat{i}$ & $\hat{j}$ so it is said to be two dimensional.

$$\boxed{|\vec{A}| = A = \sqrt{A_x^2 + A_y^2}}$$

$\because \vec{A} = A \cdot \hat{A}$

$\Rightarrow \hat{A} = \dfrac{\vec{A}}{A}$

$$\Rightarrow \boxed{\hat{A} = \frac{A_x\hat{i} + A_y\hat{j}}{\sqrt{A_x^2 + A_y^2}}}$$

e.g. $\vec{A} = 3\hat{i} + 4\hat{j}$

Find its magnitude and direction

Sol^n

$|\vec{A}| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$

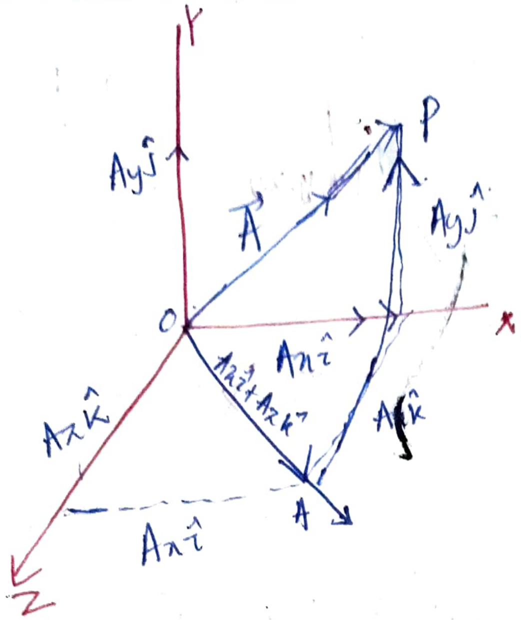
$\hat{A} = \dfrac{3\hat{i} + 4\hat{j}}{5}$

$\hat{A} = \left(\dfrac{3}{5}\right)\hat{i} + \dfrac{4}{5}\hat{j}$

H.W Question: Find magnitude and dir^n of following vecto[rs]

1. $\vec{A} = 3\hat{j} - 4\hat{k}$
2. $\vec{B} = 4\hat{j} + 2\hat{k}$
3. $\vec{C} = 2\hat{i} + 3\hat{j} - 4\hat{i}$

Vector in three dimension



The TV we are watching i.e 2d box it has a x-axis & y axis but in the other moves it comes forward towards us like X-Y-Z both are perpendicular to each other.

In Δ OAP

$$\vec{OP} = \vec{OA} + \vec{AP}$$

$$\vec{A} = A_x\hat{i} + A_z\hat{k} + A_y\hat{j}$$

$$\boxed{\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}}$$

Magnitude

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$\boxed{\hat{A} = \frac{\vec{A}}{A} = \frac{A_x\hat{i} + A_y\hat{j} + A_z\hat{k}}{\sqrt{A_x^2 + A_y^2 + A_z^2}}}$$

direction.

Prob: $\vec{A} = 3\hat{i} - 4\hat{j} + 5\hat{k}$

find $|\vec{A}|$, $\hat{A}$

Solⁿ

$$|\vec{A}| = \sqrt{3^2 + (-4)^2 + (5)^2} = \sqrt{50} = 5\sqrt{2}$$

$$\hat{A} = \frac{3\hat{i} - 4\hat{j} + 5\hat{k}}{5\sqrt{2}}$$

Vector Product :

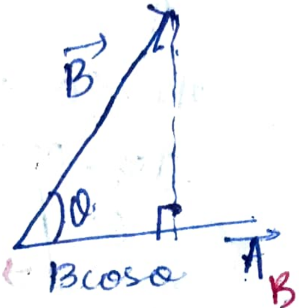
(i) Dot product or Scalar product

(2) cross product or vector product

(1) Dot product :-

$$\vec{A} \cdot \vec{B} = AB\cos\theta$$

$$= \underbrace{A(B\cos\theta)}_{\text{magnitude}}$$



A's magnitude
B's component (B cos θ's component)

→ As the result of Dot product of two vector is a scalar quantity so it is named as scalar product.

ex- $W = \vec{F} \cdot \vec{S}$

Work = force x displacement

work is a scalar because it obey dot product

1) Power = Force . velocity

$P = \vec{F} \cdot \vec{V}$

Properties of dot product :-

(i) $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$ (commutative)

(ii) $\vec{A} \cdot (\vec{B} + \vec{C}) = (\vec{A} \cdot \vec{B}) + (\vec{A} \cdot \vec{C})$ (distributive)

ex

$\vec{A} \cdot (\vec{B} \cdot \vec{C})$

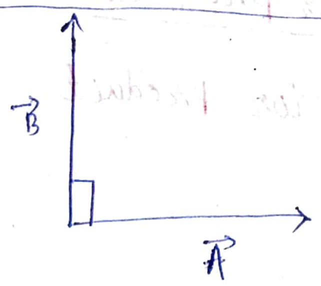
$= \underbrace{\vec{A}}_{\text{vector}} \cdot \text{scalar}$ (can't write these expression)

[Here dot product of two vector gives the scalar]

→ We can't do dot product of vector with scalar, for that dot product we need two vector.

Characteristics of dot product :

1) Perpendicular vector ($\theta = 90°$)



$\vec{A} \cdot \vec{B} = AB \cos\theta$

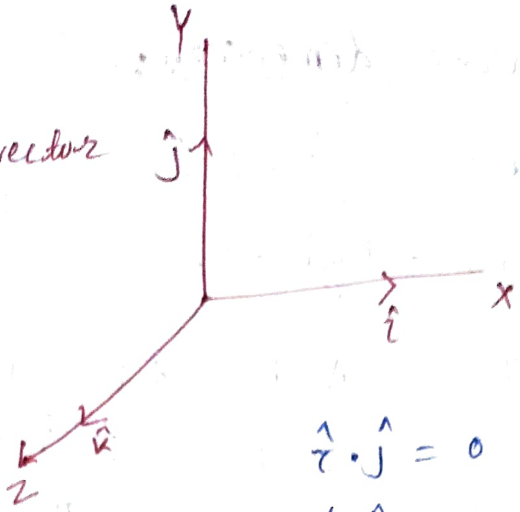
$= AB \cos 90$

$\boxed{\vec{A} \cdot \vec{B} = 0}$ → cond^n for perpendicularity

→ If the dot product between two vector is zero (0) then the angle was between two vectors ($\theta = 90°$)

For example

In orthogonal vector we learnt



$\hat{i} \perp \hat{j} \perp \hat{k}$

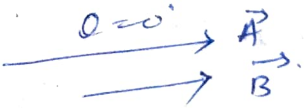
$$\hat{i} \cdot \hat{j} = 0$$
$$\hat{i} \cdot \hat{k} = 0$$
$$\hat{j} \cdot \hat{k} = 0$$

(ii) Equal vector $(\theta = 0^\circ)$

$$\vec{A} \cdot \vec{A} = AA\cos\theta$$
$$= A^2 \cos 0^\circ$$
$$= A^2$$

$$\boxed{\begin{aligned} \hat{i} \cdot \hat{i} &= |\hat{i}|^2 = 1^2 = 1 \\ \hat{j} \cdot \hat{j} &= 1 \\ \hat{k} \cdot \hat{k} &= 1 \end{aligned}}$$

cond^n for colinearity



(iii) colinear vector

$$\vec{A} \cdot \vec{B} = AB\cos\theta$$
$$= AB\cos 0^\circ$$
$$= AB$$

⑭ Vector in three dimension:-

Let $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

We know that $\vec{A}\cdot\vec{B} = AB\cos\theta$

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB}$$

$$\Rightarrow \cos\theta = \frac{A_xB_x + A_yB_y + A_zB_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}\sqrt{B_x^2 + B_y^2 + B_z^2}}$$

ex $\vec{A} = 2\hat{i} + 3\hat{j} - 4\hat{k}$

$\vec{B} = \hat{i} + 2\hat{j} + 3\hat{k}$

Find angle in between two vector.

$$\cos\theta = \frac{2+6-12}{\sqrt{4+9+16}\sqrt{1+4+9}}$$

$$\Rightarrow \cos\theta = \frac{-4}{\sqrt{29}\sqrt{14}}$$

$$\Rightarrow \cos\theta = \frac{-4}{\sqrt{391}} \Rightarrow \theta = \cos^{-1}\left(\frac{-4}{\sqrt{391}}\right)$$

Problems on dot product

Q1 $\vec{A} = 2\hat{i} + 3\hat{j} - \alpha\hat{k}$

$\vec{B} = \alpha\hat{i} + 2\hat{j} - 3\hat{k}$

For what value of α the two vectors are perpendicular with each other.

Sol For perpendicular $\vec{A}\cdot\vec{B}=0$

$$\Rightarrow 2\alpha + 6 + 3\alpha = 0$$

$$\Rightarrow 5\alpha = -6$$

$$\Rightarrow \alpha = -\frac{6}{5}$$

$$\boxed{\alpha = -1.2}$$

Q2 Find Angle in between two vectors $(\hat{i}+\hat{j})$ and $(\hat{i}-\hat{j})$

Sol^n Let $\vec{A} = \hat{i}+\hat{j}$

$\vec{B} = \hat{i}-\hat{j}$

$$\vec{A}\cdot\vec{B} = 1-1 = 0$$

$$\boxed{\vec{A}\cdot\vec{B} = 0}$$

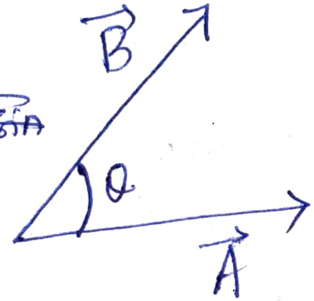
$\therefore \vec{A}\perp\vec{B}$ or $\theta = 90°$

$= \quad 5 - \frac{1}{2} = \frac{9}{2} = 4.5$

Cross product / vector product

$\cancel{\vec{A} \times \vec{B} = \vec{C} = AB\sin}$

$\hat{n}$ = normal vector



$$\vec{A} \times \vec{B} = \vec{C} = (\underbrace{AB\sin\theta}_{\text{Magnitude}})\underbrace{\hat{n}}_{\text{direcn}}$$

→ If we cross product two vector, we get the resultant as a vector quantity that have both magnitude and direcn. Here $(\vec{C})$ is the resultant.

→ The direction of $\hat{n}$ is perpendicular to the plane containing $\vec{A}$ & $\vec{B}$

→ Direction of $\hat{n}$ can be find by using right hand thumb rule.

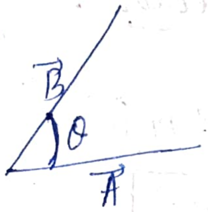
$$(\vec{B} \times \vec{A}) = -\vec{C}$$

$$\because (\vec{B} \times \vec{A}) = -(\vec{A} \times \vec{B})$$

→ We should consider smaller angle between two vector

The cross product of two vector is a vector quantity whose magnitude is equal to product of magnitude of two vectors and sine of smaller angle in between them and its direction is always perpendicular to it's plane.

$$(\vec{A} \times \vec{B}) = \vec{C}$$
$$= AB \sin\theta \, \hat{n}$$



EX Torque $(\vec{\tau}) = \vec{r} \times \vec{F}$

Properties

① $(\vec{A} \times \vec{B}) \neq (\vec{B} \times \vec{A})$

$$\because (\vec{A} \times \vec{B}) = -(\vec{B} \times \vec{A})$$

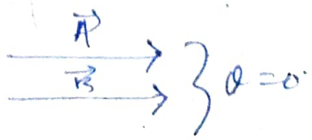
② $\vec{A} \times (\vec{B} + \vec{C}) = (\vec{A} \times \vec{B}) + (\vec{A} \times \vec{C})$

Characteristics of cross product:-

① colinear vector

ⓐ Parallel vector $(\theta = 0°)$

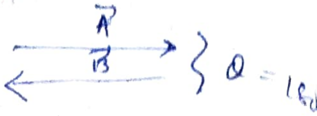
$$\vec{A} \times \vec{B} = AB \sin\theta \, \hat{n}$$
$$= AB \sin 0°$$
$$\boxed{\vec{A} \times \vec{B} = \vec{0}}$$



e.g. $\hat{i} \times \hat{i} = 0$
$\hat{j} \times \hat{j} = 0$
$\hat{k} \times \hat{k} = 0$

(b) Antiparallel vector :- ($\theta = 180°$)

$$\vec{A}\times\vec{B} = AB\sin\theta\,\hat{n}$$



$$\Rightarrow \boxed{\vec{A}\times\vec{B} = \vec{0}}$$

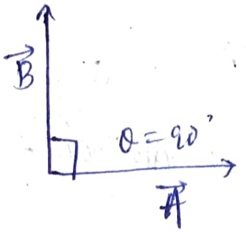
It is the condition for colinearity.

The cross product of two colinear vector is always a zero vector. This condition is known as condition for colinearity.

Perpendicular vector :

$$(\vec{A}\times\vec{B}) = AB\sin\theta\,\hat{n}$$

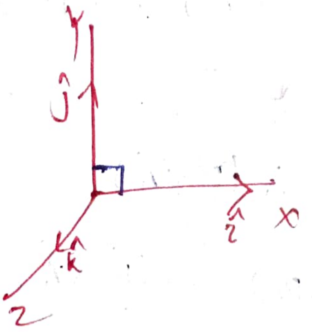
$$\Rightarrow \boxed{\vec{A}\times\vec{B} = AB\,\hat{n}}$$



e.g.

$\hat{i}\times\hat{j} = 1\times1\,\hat{n} = \hat{n}$

$\hat{i}\times\hat{j} = \hat{k}$



$\hat{j}\times\hat{k} = (1)(1)\,\hat{n}$
$= \hat{n}$

$\hat{j}\times\hat{k} = \hat{i}$

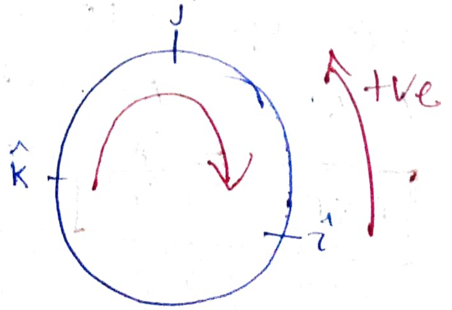
$\hat{k}\times\hat{i} = (1)(1)\,\hat{n} = \hat{n}$

$\hat{k}\times\hat{i} = \hat{j}$

→ $\hat{j}\times\hat{i} = -\hat{k}$

→ $\hat{k}\times\hat{j} = \hat{i}$

$\hat{i}\times\hat{k} = -\hat{j}$



Vector in three dimension / Three dimensional vector

Let $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$$\vec{A}\times\vec{B} = \begin{array}{ccc} (+) & (-) & (+) \end{array}$$
$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= +\hat{i}\begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} - \hat{j}\begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} + \hat{k}\begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix}$$

$$= \hat{i}(A_yB_z - A_zB_y) - \hat{j}(A_xB_z - A_zB_x) + \hat{k}(A_xB_y - A_yB_x)$$

e.g

$\vec{A} = 3\hat{i} + 2\hat{j} + 4\hat{k}$

$\vec{B} = 2\hat{i} + 3\hat{j} + 3\hat{k}$

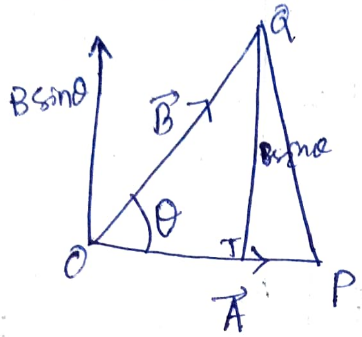
$$\vec{A}\times\vec{B} = \hat{i}(6-12) - \hat{j}(9-8) + \hat{k}(9-4)$$

$$= -6\hat{i} - \hat{j} + 5\hat{k}$$

Application of cross product

→ By the help of cross product we can find out the area of parallelogram, triangle.

Area of triangle :-



Area of the triangle
$= \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times OP \times QT$

$= \frac{1}{2} A \times B\sin\theta$

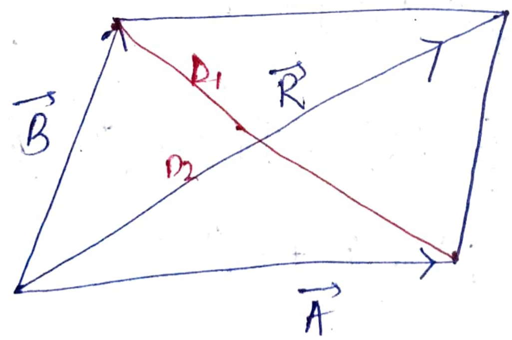
$= \frac{1}{2} AB\sin\theta$

$$\boxed{= \frac{1}{2} |\vec{A} \times \vec{B}|}$$

$(\vec{A} \times \vec{B}) = AB\sin\theta\,\hat{n}$

$|\vec{A} \times \vec{B}| = AB\sin\theta$

Area of parallelogram :-



→ Parallelogram made of two ~~vectors~~ triangles if Area of one (1) triangle $= \frac{1}{2} |\vec{A} \times \vec{B}|$ then

Area of parallelogram $= |\vec{A} \times \vec{B}| = 2 \times \frac{1}{2} |\vec{A} \times \vec{B}|$

$$\boxed{= |\vec{A} \times \vec{B}|}$$

When diagonal is given then the
Area of Parallelogram $= |\vec{B_1} \times \vec{B_2}|$

Problems on cross product

Q-1 Let $\vec{A} = 2\hat{i} + 3\hat{j} - 4\hat{k}$
$\vec{B} = 3\hat{i} - 2\hat{j} + 6\hat{k}$

If $\vec{C} = (\vec{A} \times \vec{B})$ then find direction.

Sol^n $\vec{A} = 2\hat{i} + 3\hat{j} - 4\hat{k}$
$\vec{B} = 3\hat{i} - 2\hat{j} + 6\hat{k}$

$(\vec{A} \times \vec{B}) = \vec{C} = \hat{i}(18 - 8) - \hat{j}(12 + 12) + \hat{k}(-4 - 9)$

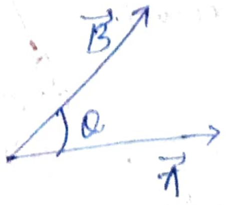
$= 10\hat{i} - 24\hat{j} - 13\hat{k}$

$$\hat{C} = \frac{\vec{C}}{|\vec{C}|} = \frac{10\hat{i} - 24\hat{j} - 13\hat{k}}{\sqrt{100 + 576 + 169}}$$

Q2 Find the value of $(\vec{A} \times \vec{B}) \cdot \vec{A}$

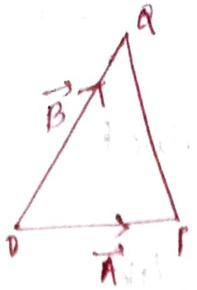
Ans $(\vec{A} \times \vec{B}) \cdot \vec{A}$
Let $\vec{C} = (\vec{A} \times \vec{B})$

So $\vec{C} \cdot \vec{A} = CA\cos\theta$



→ If we cross product $(\vec{A} \times \vec{B})$ we get $\vec{C}$ then dir^n of $\vec{C}$ it is ⊥r to both $\vec{A}$ and $\vec{B}$ i.e
$\theta = 90°$ $\vec{C} \cdot A = CA\cos 90$ $(\because \theta = 90°)$
$= 0$

1. Q-3



here $\vec{A} = \hat{i} + \hat{j} + 3\hat{k}$

$\vec{B} = 2\hat{i} + 3\hat{j} - \hat{k}$

Find Area of triangle

sol^n

$$\text{Area} = \frac{1}{2}|\vec{A} \times \vec{B}|$$

$$\vec{A} \times \vec{B} = \hat{i}(-1-9) - \hat{j}(-1-6) + \hat{k}(3-2)$$

$$\Rightarrow (\vec{A} \times \vec{B}) = -10\hat{i} + 7\hat{j} + \hat{k}$$

$$|\vec{A} \times \vec{B}| = \sqrt{100 + 49 + 1}$$

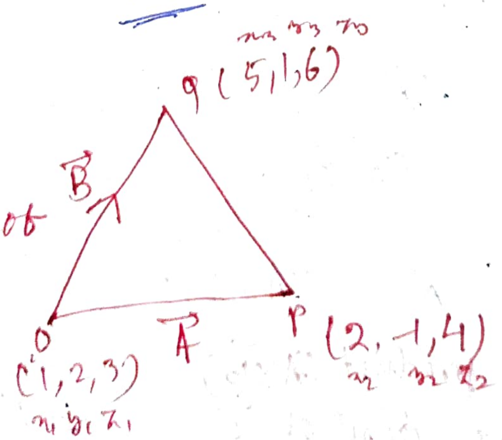
$$= \sqrt{150}$$

$$\text{Area} = \frac{1}{2} \times \sqrt{150}$$

Ans

Q4

Find Area of this vector.



$$\vec{A} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

$$\vec{B} = (x_3 - x_1)\hat{i} + (y_3 - y_1)\hat{j} + (z_3 - z_1)\hat{k}$$

then $\text{Area} = \frac{1}{2}|\vec{A} \times \vec{B}|$

Find co-ordinate then cross product them

Questions on vector

Q. If $|\vec{A} \times \vec{B}| = \sqrt{3}(\vec{A} \cdot \vec{B})$
then find $|\vec{A} + \vec{B}|$

Sol^n $|\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$

$|\vec{A} \times \vec{B}| = \sqrt{3}(\vec{A} \cdot \vec{B})$

$\Rightarrow AB\sin\theta = \sqrt{3}\,AB\cos\theta$

$\Rightarrow \tan\theta = \sqrt{3}$

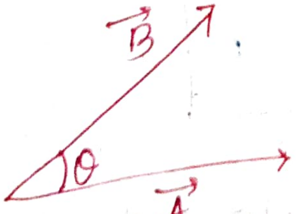
$\Rightarrow \tan\theta = \tan 60$

$\Rightarrow \boxed{\theta = 60^\circ}$

$|\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - 2AB \cdot \frac{1}{2}}$ $\left[\because \theta = 60^\circ,\ \cos 60 = \frac{1}{2}\right]$

$= \sqrt{A^2 + B^2 - AB}$

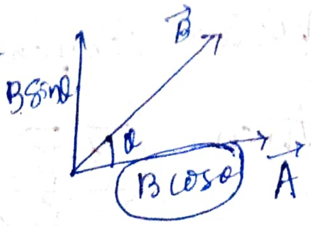
Note



① Find scalar component of $\vec{B}$ along $\vec{A}$

② Find vector component of $\vec{B}$ along $\vec{A}$

Sol^n (a) Scalar component $= \cancel{B\cos\theta}$
$= B$



Let me resolve $\vec{B}$ then i will find two component $B\cos\theta$ & $B\sin\theta$ where as $B\cos\theta$ along $\vec{A}$.

1. Scalar component

$$= B\cos\theta$$

$$= B\left(\frac{\vec{A}\cdot\vec{B}}{AB}\right)$$

$$= \left(\frac{\vec{A}\cdot\vec{B}}{A}\right)$$

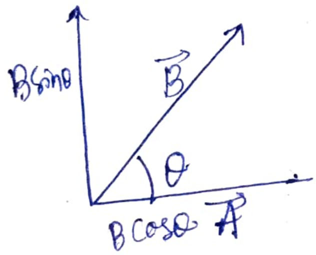
$$\left[\ \vec{A}\cdot\vec{B} = AB\cos\theta \qquad \frac{\vec{A}\cdot\vec{B}}{A\cdot B} = \cos\theta\ \right]$$

Formula for scalar component of $\vec{B}$ along $\vec{A}$ = $\boxed{\frac{\vec{A}\cdot\vec{B}}{A}}$

→ If it is ask find scalar component of $\vec{A}$ along $\vec{B}$

then $\boxed{\frac{\vec{B}\cdot\vec{A}}{B}}$ ✓

(2) Soln



Though vector has both magnitude and direction so we have to find both dirn & magnitude

Vector component of $\vec{B}$ along $\vec{A}$

$$= \underbrace{(B\cos\theta)}_{M}\ \underbrace{\hat{A}}_{D}$$

$$= B\cdot\left(\frac{\vec{A}\cdot\vec{B}}{AB}\right)\hat{A}$$

$$\boxed{= \left(\frac{\vec{A}\cdot\vec{B}}{A}\right)\hat{A}}$$

Question Find scalar component of $\vec{X}$ along $\vec{Y}$

$$= \frac{\vec{X}\cdot\vec{Y}}{Y}$$

If it is given vector component of $\vec{X}$ along $\vec{Y}$

then $\left(\frac{\vec{X}\cdot\vec{Y}}{Y}\right)\hat{Y}$

④ Find vector component $\vec{P}$ along $\vec{q}$

$$= \left(\frac{\vec{P}\cdot\vec{Q}}{Q}\right)\hat{q}$$

Problem - I

Let $\vec{A} = 2\hat{i} + 3\hat{j} - 4\hat{k}$

$\vec{B} = 3\hat{i} - \hat{j} - 4\hat{k}$

Find an scalar component of $\vec{A}$ along $\vec{B}$

Ans

$$\frac{\vec{A}\cdot\vec{B}}{B} =$$

$$\vec{A}\cdot\vec{B} = (2\hat{i} + 3\hat{j} - 4\hat{k})\cdot(3\hat{i} - \hat{j} - 4\hat{k})$$

$$= \frac{6 - 3 + 16}{\sqrt{3^2 + 1^2 + 4^2}} = \frac{19}{\sqrt{26}}$$

⑤ Find vector component of $\vec{A}$ along $\vec{B}$

$$= \left(\frac{\vec{A}\cdot\vec{B}}{B}\right)\hat{B}$$

$$= \frac{19}{\sqrt{26}} \times \frac{\vec{B}}{B}$$

$$= \frac{19}{\sqrt{26}} \frac{3\hat{i} - \hat{j} - 4\hat{k}}{\sqrt{26}}$$

$$= \frac{19}{26}(3\hat{i} - \hat{j} - 4\hat{k})$$