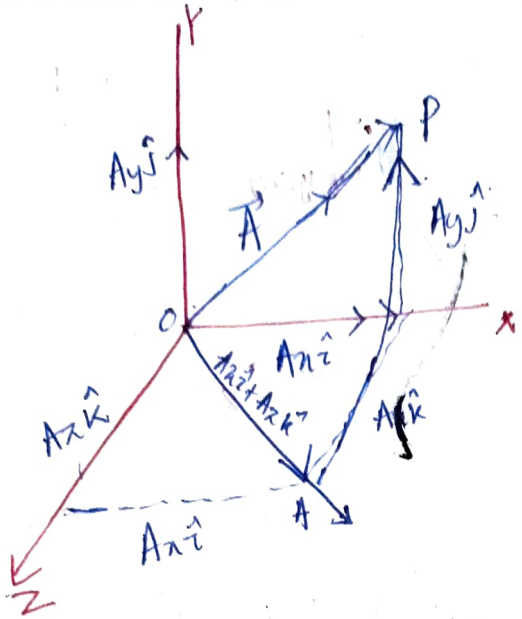
Vector in three dimension

The TV we are watching i.e 2d box it has a x-axis & y axis but on the other hand it comes towards us like x-y-z both are perpendicular to each other.



on ΔOAP

$$\vec{OP} = \vec{OA} + \vec{AP}$$

$$\Rightarrow \vec{A} = A_x\hat{i} + A_z\hat{k} + A_y\hat{j}$$

$$\boxed{\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}}$$ — magnitude

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$\hat{A} = \frac{\vec{A}}{A} = \frac{A_x\hat{i} + A_y\hat{j} + A_z\hat{k}}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$ — direction.

Prob $\vec{A} = 3\hat{i} - 4\hat{j} + 5\hat{k}$

find $|\vec{A}|$, $\hat{A}$

Sol^n

$$|\vec{A}| = \sqrt{3^2 + (-4)^2 + (5)^2} = \sqrt{50} = 5\sqrt{2}$$

$$\hat{A} = \frac{3\hat{i} - 4\hat{j} + 5\hat{k}}{5\sqrt{2}}$$

Vector product :

(i) Dot product or Scalar product

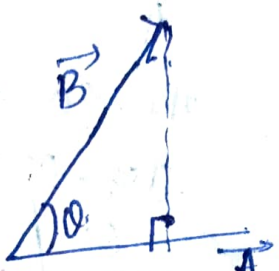
(2) cross product or vector product

(1) Dot product :-

$$\vec{A} \cdot \vec{B} = AB\cos\theta$$

$$= A(B\cos\theta)$$

magnitude



A → magnitude

$B\cos\theta$ → component

→ As the result of Dot product of two vector is a scalar quantity so it is named as Scalar product.

ex- $W = \vec{F} \cdot \vec{S}$

Work = force x displacement

Work is a scalar because it obey dot product

Power = Force . velocity

$$P = \vec{F} \cdot \vec{V}$$

Properties of dot product :-

(i) $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$ (commutative)

(ii) $\vec{A} \cdot (\vec{B} + \vec{C}) = (\vec{A} \cdot \vec{B}) + (\vec{A} \cdot \vec{C})$ (distributive)

Ex

$\vec{A} \cdot (\vec{B} \cdot \vec{C})$

$= \vec{A} \cdot$ Scalar

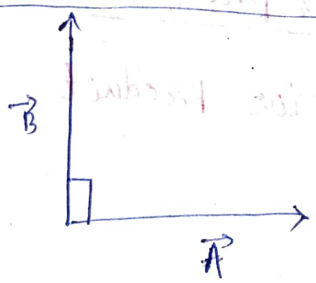
vector (can't write these expression)

Here dot product of two vector gives the scalar

→ we can't do dot product of vector with scalar. for that dot product we need two vector.

Characteristics of dot product :

1) Perpendicular vector ($\theta = 90°$)



$\vec{A} \cdot \vec{B} = AB \cos\theta$

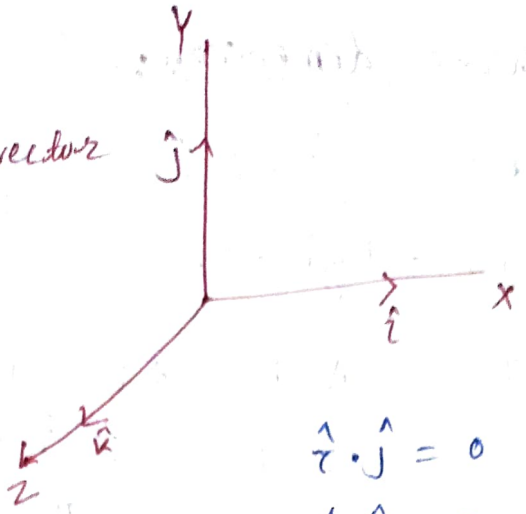
$= AB \cos 90$

$\boxed{\vec{A} \cdot \vec{B} = 0}$ → cond^n for perpendicularity

→ If the dot product between two vector is zero(0) then the angle was between two vectors ($\theta = 90°$).

For example

an orthogonal vector we learnt



$\vec{r} = 1\hat{i} + 1\hat{j} + 1\hat{k}$

$\hat{i} \cdot \hat{j} = 0$

$\hat{i} \cdot \hat{k} = 0$

$\hat{j} \cdot \hat{k} = 0$

(ii) Equal vector ($\theta = 0^\circ$)

$\vec{A} \cdot \vec{A} = AA\cos\theta$

$= A^2 \cos 0^\circ$

$= A^2$

$\hat{i} \cdot \hat{i} = |\hat{i}|^2 = 1^2 = 1$

$\hat{j} \cdot \hat{j} = 1$

$\hat{k} \cdot \hat{k} = 1$

cond^n for colinearity

(iii) colinear vector

$\theta = 0^\circ$ → $\vec{A}$, $\vec{B}$

$\vec{A} \cdot \vec{B} = AB\cos\theta$

$= AB\cos 0^\circ$

$= AB$

(N) Vector in three dimension :-

Let $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

We know that $\vec{A}.\vec{B} = AB\cos\theta$

$$\cos\theta = \frac{\vec{A}.\vec{B}}{AB}$$

$$\Rightarrow \cos\theta = \frac{A_xB_x + A_yB_y + A_zB_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}\sqrt{B_x^2 + B_y^2 + B_z^2}}$$

Ex $\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{B} = \hat{i} + 2\hat{j} + 3\hat{k}$

Find angle in between two vector.

$$\cos\theta = \frac{2 + 6 - 12}{\sqrt{4+9+16}\sqrt{1+4+9}}$$

$$\Rightarrow \cos\theta = \frac{-4}{\sqrt{29}\sqrt{14}}$$

$$\Rightarrow \cos\theta = \frac{-4}{\sqrt{391}} \Rightarrow \theta = \cos^{-1}\left(\frac{-4}{\sqrt{391}}\right)$$

Problems on dot product

Q1 $\vec{A} = 2\hat{i} + 3\hat{j} - \alpha\hat{k}$

$\vec{B} = \alpha\hat{i} + 2\hat{j} - 3\hat{k}$

For what value of α the two vectors are perpendicular with each other.

Sol" For perpendiculars $\vec{A} \cdot \vec{B} = 0$

$$\Rightarrow 2\alpha + 6 + 3\alpha = 0$$

$$\Rightarrow 5\alpha = -6$$

$$\Rightarrow \alpha = -\frac{6}{5}$$

$$\boxed{\alpha = -1.2}$$

12 Find Angle in between two vectors $(\hat{i} + \hat{j})$ and $(\hat{i} - \hat{j})$

Sol"

Let $\vec{A} = \hat{i} + \hat{j}$

$\vec{B} = \hat{i} - \hat{j}$

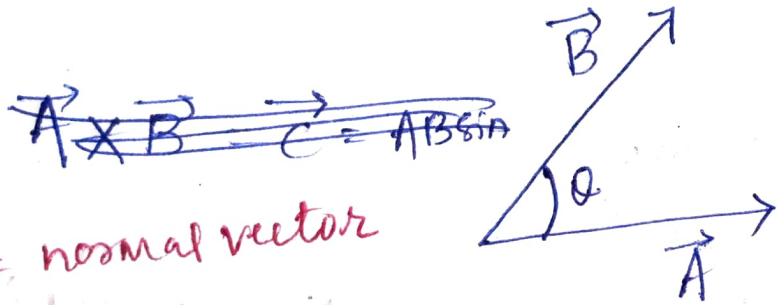
$\vec{A} \cdot \vec{B} = 1 - 1 = 0$

$$\boxed{\vec{A} \cdot \vec{B} = 0}$$

$\therefore \vec{A} \perp \vec{B}$ or $\theta = 90^\circ$

$= 5 - \frac{1}{2} = \frac{9}{2} = 4.5$

Cross product / vector product



$$\vec{A} \times \vec{B} = \vec{C} = (AB\sin\theta)\hat{n}$$ — direction

(Magnitude)

→ If we cross product two vector, we get the resultant as a vector quantity that have both Magnitude and direction. Here $(\vec{C})$ is the resultant

→ The direction of $\hat{n}$ is perpendicular to the plane containing $\vec{A}$ & $\vec{B}$

→ Direction of $\hat{n}$ can be find by using right hand thumb rule.

$$(\vec{B} \times \vec{A}) = -\vec{C}$$

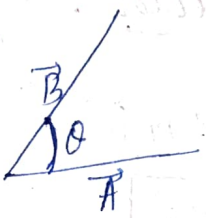
$$\because (\vec{B} \times \vec{A}) = -(\vec{A} \times \vec{B})$$

→ We should consider smaller angle between two vector

The cross product of two vector is a vector quantity whose magnitude is equal to product of magnitude of two vectors and sine of smaller angle in between them and its direction is always perpendicular to it's plane.

$$(\vec{A} \times \vec{B}) = \vec{C}$$

$$= AB \sin\theta \hat{n}$$



EX Torque $(\vec{\tau}) = \vec{r} \times \vec{F}$

Properties ✓

(1) $(\vec{A} \times \vec{B}) \neq (\vec{B} \times \vec{A})$

$$\because (\vec{A} \times \vec{B}) = -(\vec{B} \times \vec{A})$$

(2) $\vec{A} \times (\vec{B} + \vec{C}) = (\vec{A} \times \vec{B}) + (\vec{A} \times \vec{C})$

Characteristics of cross product :-

(1) colinear vector

(a) Parallel vector ($\theta = 0^\circ$)

$$\vec{A} \times \vec{B} = AB \sin\theta \hat{n}$$

$$= AB \sin 0$$

$$\boxed{\vec{A} \times \vec{B} = \vec{0}}$$

$\vec{A}$ →
$\vec{B}$ → } $\theta = 0^\circ$

e.g $\hat{i} \times \hat{i} = 0$

$\hat{j} \times \hat{j} = 0$

$\hat{k} \times \hat{k} = 0$

(b) **Antiparallel vector** :- $(\theta = 180^\circ)$.

$\vec{A} \times \vec{B} = AB \sin\theta \hat{n}$

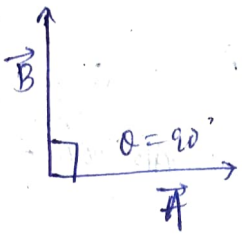
$\Rightarrow \boxed{\vec{A} \times \vec{B} = \vec{0}}$ it is the condition for colinearity.

The cross product of two colinear vector is always a zero vector. This condition is known as condition for colinearity.

Perpendicular vector :

$(\vec{A} \times \vec{B}) = AB \sin\theta \hat{n}$

$\Rightarrow \boxed{\vec{A} \times \vec{B} = AB\hat{n}}$



e.g $\hat{i} \times \hat{j} = |\times| \hat{n} = \hat{n}$

$\hat{i} \times \hat{j} = \hat{k}$

$\hat{j} \times \hat{k} = (1)(1)\hat{n}$

$= \hat{n}$

$\hat{j} \times \hat{k} = \hat{i}$

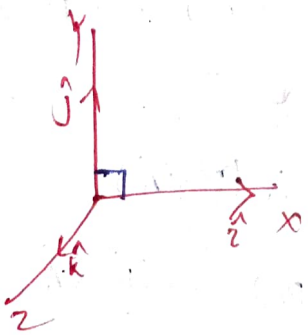
$\hat{k} \times \hat{i} = (1)(1)\hat{n} = \hat{n}$

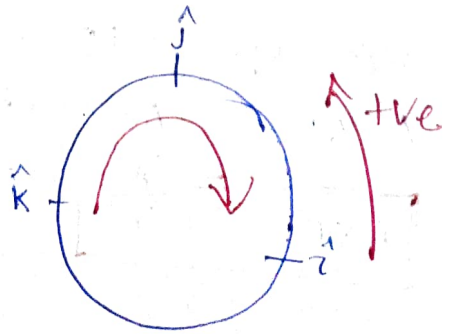
$\hat{k} \times \hat{i} = \hat{j}$

$\Rightarrow \hat{j} \times \hat{i} = -\hat{k}$

$\Rightarrow \hat{k} \times \hat{j} = \hat{i}$

$\hat{i} \times \hat{k} = -\hat{j}$





Vector in three dimension / Three dimensional vector

Let $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

(+) (−) (+)

$$= +\hat{i}\begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} - \hat{j}\begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} + \hat{k}\begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix}$$

$$= \hat{i}(A_yB_z - A_zB_y) - \hat{j}(A_xB_z - A_zB_x) + \hat{k}(A_xB_y - A_yB_x)$$

e.g

$\vec{A} = 3\hat{i} + 2\hat{j} + 4\hat{k}$

$\vec{B} = 2\hat{i} + 3\hat{j} + 3\hat{k}$

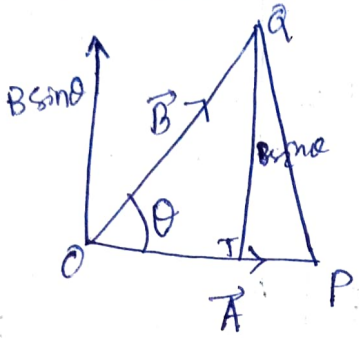
$\vec{A} \times \vec{B} = \hat{i}(6-12) - \hat{j}(9-8) + \hat{k}(9-4)$

$= -6\hat{i} - \hat{j} + 5\hat{k}$

Application of cross product

→ By the help of cross product we can find out the area of parallelogram, triangle.

Area of triangle :-



$(\vec{A} \times \vec{B}) = AB \sin\theta \hat{n}$

$|\vec{A} \times \vec{B}| = AB \sin\theta$

Area of the triangle

$= \frac{1}{2} \times$ base $\times$ height

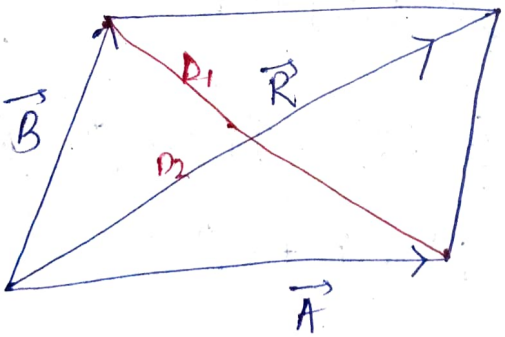
$= \frac{1}{2} \times OP \times QT$

$= \frac{1}{2} A \times B \sin\theta$

$= \frac{1}{2} AB \sin\theta$

$= \frac{1}{2} |\vec{A} \times \vec{B}|$

Area of parallelogram :-



→ Parallelogram made of two ~~vectors~~ triangles if Area of one (1) tringle $= \frac{1}{2} |\vec{A} \times \vec{B}|$ then

Area of Parallelogram $= |\vec{A} \times \vec{B}| = 2 \times \frac{1}{2} |\vec{A} \times \vec{B}|$

$= |\vec{A} \times \vec{B}|$

When diagonal is given then the

Area of parallelogram = $|\vec{B}_1 \times \vec{B}_2|$

Problems on cross product

Q-1 Let $\vec{A} = 2\hat{i} + 3\hat{j} - 4\hat{k}$

$\vec{B} = 3\hat{i} - 2\hat{j} + 6\hat{k}$

If $\vec{C} = (\vec{A} \times \vec{B})$ then find direction.

Soln $\vec{A} = 2\hat{i} + 3\hat{j} - 4\hat{k}$

$\vec{B} = 3\hat{i} - 2\hat{j} + 6\hat{k}$

$(\vec{A} \times \vec{B}) = \vec{C} = \hat{i}(18 - 8) - \hat{j}(12 + 12) + \hat{k}(-4 - 9)$

$= 10\hat{i} - 24\hat{j} - 13\hat{k}$

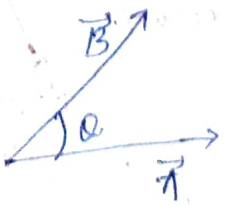
$\hat{C} = \dfrac{\vec{C}}{|\vec{C}|} = \dfrac{10\hat{i} - 24\hat{j} - 13\hat{k}}{\sqrt{100 + 576 + 169}}$

Q2 Find the value of $(\vec{A} \times \vec{B}) \cdot \vec{A}$

Ans $(\vec{A} \times \vec{B}) \cdot \vec{A}$

Let $\vec{C} = (\vec{A} \times \vec{B})$

So $\vec{C} \cdot \vec{A} = CA\cos\theta$

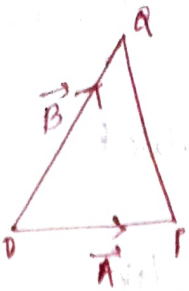


→ If we cross product $(\vec{A} \times \vec{B})$ we get $\vec{C}$ then dir^n of $\vec{C}$ is ⊥r to both $\vec{A}$ and $\vec{B}$ i.e $\theta = 90$

$\vec{C} \cdot A = CA\cos 90 \quad (\because \theta = 90)$

$= 0$

Q-3



here $\vec{A} = \hat{i} + \hat{j} + 3\hat{k}$

$\vec{B} = 2\hat{i} + 3\hat{j} - \hat{k}$

Find Area of triangle

Soln

$\text{Area} = \frac{1}{2} |\vec{A} \times \vec{B}|$

$\vec{A} \times \vec{B} = \hat{i}(-1-9) - \hat{j}(-1-6) + \hat{k}(3-2)$

$\Rightarrow (\vec{A} \times \vec{B}) = -10\hat{i} + 7\hat{j} + \hat{k}$

$|\vec{A} \times \vec{B}| = \sqrt{100 + 49 + 1}$

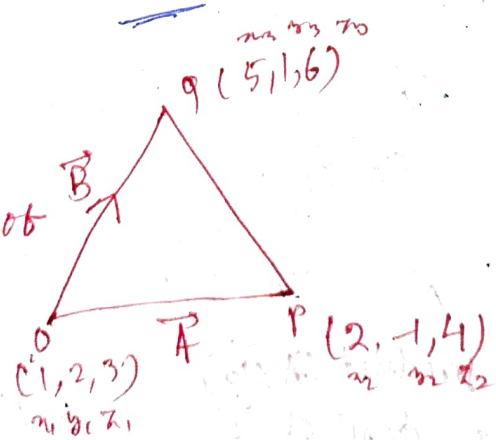
$= \sqrt{150}$

$\text{Area} = \frac{1}{2} \times \sqrt{150}$

Ans

Q4

Find Area of this vector.



$\vec{A} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

$\vec{B} = (x_3 - x_1)\hat{i} + (y_3 - y_1)\hat{j} + (z_3 - z_1)\hat{k}$

then Area $= \frac{1}{2} |\vec{A} \times \vec{B}|$

Find co-ordinate then cross product then

Questions on vector

Q If $|\vec{A} \times \vec{B}| = \sqrt{3}(\vec{A} \cdot \vec{B})$ then find $|\vec{A} + \vec{B}|$

Sol^n $|\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$

$|\vec{A} \times \vec{B}| = \sqrt{3}(\vec{A} \cdot \vec{B})$

$\Rightarrow AB\sin\theta = \sqrt{3}AB\cos\theta$

$\Rightarrow \tan\theta = \sqrt{3}$

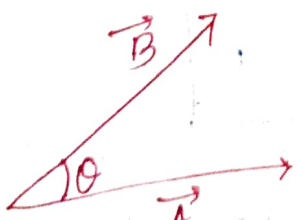
$\Rightarrow \tan\theta = \tan 60$

$\Rightarrow \boxed{\theta = 60}$

$|\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - 2AB \cdot \frac{1}{2}}$ $\left[\because \theta = 60, \cos 60 = \frac{1}{2}\right]$

$= \sqrt{A^2 + B^2 - AB}$

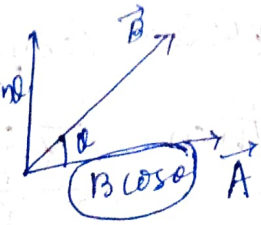
Note



① Find scalar component of $\vec{B}$ along $\vec{A}$

② Find vector component of $\vec{B}$ along $\vec{A}$

Sol^n ① Scalar component $= B\cos\theta$.



Let me resolve $\vec{B}$ then I will find two component $B\cos\theta$ & $B\sin\theta$ where as $B\cos\theta$ along $\vec{A}$.

Scalar component

$= B\cos\theta$ $\quad \because \left[\vec{A}\cdot\vec{B} = AB\cos\theta \;\Rightarrow\; \frac{\vec{A}\cdot\vec{B}}{AB} = \cos\theta\right]$

$= B\left(\frac{\vec{A}\cdot\vec{B}}{AB}\right)$

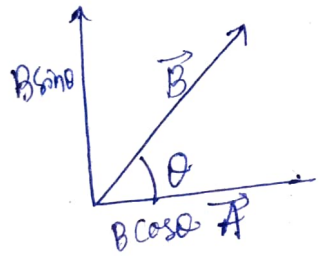
$= \left(\frac{\vec{A}\cdot\vec{B}}{A}\right)$

Formula for scalar component of $\vec{B}$ along $\vec{A} = \boxed{\frac{\vec{A}\cdot\vec{B}}{A}}$

→ If it is ask find scalar component of $\vec{A}$ along $\vec{B}$ then $\boxed{\frac{\vec{B}\cdot\vec{A}}{B}}$ ✓

(2)

Sol'n



[Though vector has both magnitude and direction so we have to find both dirⁿ & magnitude.]

Vector component of $\vec{B}$ along $\vec{A}$

$= (B\cos\theta)\,\hat{A}$

$= B\cdot\left(\frac{\vec{A}\vec{B}}{AB}\right)\hat{A}$

$\boxed{= \left(\frac{\vec{A}\cdot\vec{B}}{A}\right)\hat{A}}$

Question Find scalar component of $\vec{x}$ along $\vec{y}$

$= \frac{\vec{x}\cdot\vec{y}}{y}$

If it is given vector component of $\vec{x}$ along $\vec{y}$ then $\left(\frac{\vec{x}\cdot\vec{y}}{y}\right)\hat{y}$

(4) Find vector component $\vec{P}$ along $\vec{Q}$

$$= \left(\frac{\vec{P}\cdot\vec{Q}}{Q}\right)\hat{Q}$$

Problem 1

Let $\vec{A} = 2\hat{i} + 3\hat{j} - 4\hat{k}$

$\vec{B} = 3\hat{i} - \hat{j} - 4\hat{k}$

Find a) Scalar component of $\vec{A}$ along $\vec{B}$

Soln

$$\frac{\vec{A}\cdot\vec{B}}{B} =$$

$$\vec{A}\cdot\vec{B} = (2\hat{i} + 3\hat{j} - 4\hat{k}) \cdot (3\hat{i} - \hat{j} - 4\hat{k})$$

$$= \frac{6 - 3 + 16}{\sqrt{3^2 + 1^2 + 4^2}} = \frac{19}{\sqrt{26}}$$

(5) Find vector component of $\vec{A}$ along $\vec{B}$

$$= \left(\frac{\vec{A}\cdot\vec{B}}{B}\right)\hat{B}$$

$$= \frac{19}{\sqrt{26}} \times \frac{\vec{B}}{B}$$

$$= \frac{19}{\sqrt{26}} \frac{3\hat{i} - \hat{j} - 4\hat{k}}{\sqrt{26}}$$

$$= \frac{19}{26}(3\hat{i} - \hat{j} - 4\hat{k})$$